On periodic signals in stochastic Hodgkin-Huxley models

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We consider a stochastic Hodgkin-Huxley model where dendritic input —modelled as an autonomous SDE which depends on a deterministic $T$-periodic signal $t \rightarrow S(t)$ encoded in its drift— is the only source of noise. This amounts to a 5d random system driven by 1d Brownian motion. We do have criteria to prove positive Harris recurrence (ergodicity) for systems of this type ([2], [3], [1]). As a consequence, we dispose of strong laws of large numbers for the system. In particular, we can describe the spiking activity of the neuron in the long run using strong laws of large numbers.

Let $\tau_n$ denote the beginning of the $n$-th spike. Whereas successive interspike times $\xi_n := \tau_{n+1} - \tau_n$ have no reason to be independent, there is a Glivenko-Cantelli theorem ([2]) for the sequence $(\xi_n)_{n \geq 1}$: empirical distribution functions $\hat{F}_n$ converge as $n \rightarrow \infty$ to some honest limit distribution function $F$. This limit $F$ characterizes the spiking behaviour of the neuron in the long run. It depends on the modelization of the dendritic input, in particular on the signal $t \rightarrow S(t)$ encoded in its drift.

We are interested in statistical inference on the unobserved deterministic signal $t \rightarrow S(t)$, assuming that the Hodgkin Huxley neuron can be observed over a long time interval.

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References:

