

Dynamics under location uncertainty and other energy-related stochastic subgrid schemes

V. Resseguier
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Motivations

- More rigorously identified subgrid dynamics effects
 - Quantification of modeling errors (UQ)
-  Ensemble forecasts and data assimilation

Contents

- Models under location uncertainty (LU)
- Some parameterization of the models under location uncertainty
- A new energy-budget-based stochastic scheme: WaveHyperv
- Numerical comparisons

Part I

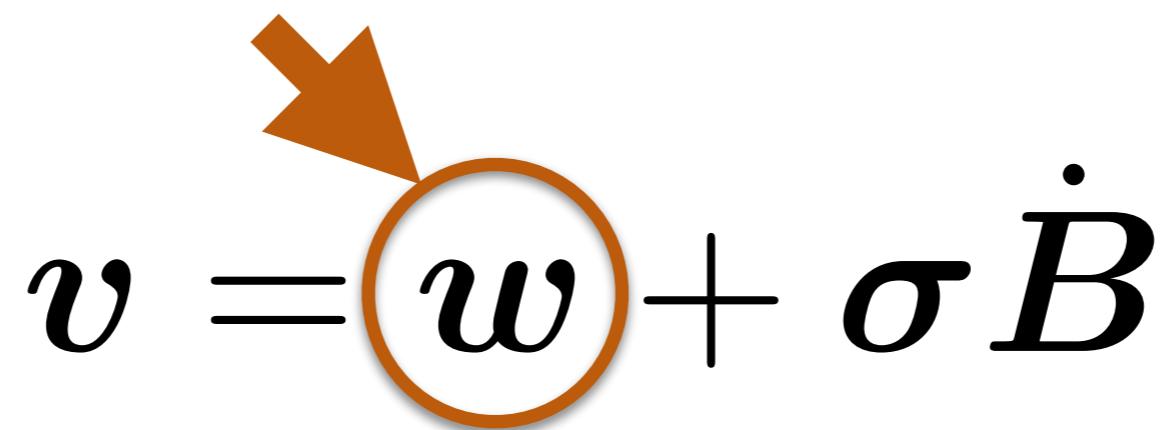
Models under location uncertainty (LU)

LU :Adding
random velocity

$$v = w + \sigma \dot{B}$$

LU :Adding random velocity

Resolved
large scales

$$v = w + \sigma \dot{B}$$


LU :Adding random velocity

Resolved
large scales

White-in-time
small scales

$$v = w + \sigma \dot{B}$$

The diagram illustrates the decomposition of velocity v into two components. On the left, the symbol v is shown with an orange arrow pointing towards it, indicating the total velocity. To its right is the equation $v = w + \sigma \dot{B}$. The term w is enclosed in an orange circle and has an orange arrow pointing towards it, labeled "Resolved large scales". The term $\sigma \dot{B}$ is enclosed in a purple circle and has a purple arrow pointing towards it, labeled "White-in-time small scales".

Large scales:

$$\mathbf{w}$$

Small scales:

$$\sigma \dot{\mathbf{B}}$$

Variance tensor:

$$\mathbf{a} = \mathbf{a}(x, x) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\mathbf{B} (\boldsymbol{\sigma} d\mathbf{B})^T\}}{dt}$$

LU :Adding random velocity

Resolved
large scales

White-in-time
small scales

$$\mathbf{v} = \mathbf{w} + \sigma \dot{\mathbf{B}}$$

The diagram illustrates the decomposition of velocity \mathbf{v} into two components. On the left, the equation $\mathbf{v} = \mathbf{w} + \sigma \dot{\mathbf{B}}$ is shown. The term \mathbf{w} is enclosed in an orange circle and has an orange arrow pointing to it from the text "Resolved large scales" above. The term $\sigma \dot{\mathbf{B}}$ is enclosed in a purple circle and has a purple arrow pointing to it from the text "White-in-time small scales" to its right.

Large scales:

$$w$$

Small scales:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

LU :Adding random velocity

Resolved
large scales

White-in-time
small scales

$$v = w + \sigma \dot{B}$$

References : Mikulevicius & Rozovskii, 2004 Flandoli, 2011

Memin, 2014
Resseguier et al. 2017 a, b, c
Cai et al. 2017
Chapron et al. 2018
Yang & Memin 2019
Resseguier et al. 2019 a,b

Holm, 2015
Holm and Tyranowski, 2016
Arnaudon et al. 2017
Cotter and al. 2017

Crisan et al., 2017
Gay-Balmaz & Holm 2017
Cotter and al. 2018 a, b
Cotter and al. 2019

Large scales:
 w

Small scales:
 $\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

LU :Adding random velocity

Resolved
large scales

White-in-time
small scales

$$v = w + \sigma \dot{B}$$

References : Mikulevicius & Rozovskii, 2004 Flandoli, 2011

LU

- Memin**, 2014
Resseguier et al. 2017 a, b, c
Cai et al. 2017
Chapron et al. 2018
Yang & Memin 2019
Resseguier et al. 2019 a,b

SALT

- Holm**, 2015
Holm and Tyranowski, 2016
Arnaudon et al. 2017
Cotter and al. 2017

- Crisan et al., 2017
Gay-Balmaz & Holm 2017
Cotter and al. 2018 a, b
Cotter and al. 2019

Large scales:

$$\boldsymbol{w}$$

Small scales:

$$\sigma \dot{\boldsymbol{B}}$$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Large scales:

$$\boldsymbol{w}$$

Small scales:

$$\sigma \dot{\boldsymbol{B}}$$

Variance
tensor:

$$\boldsymbol{a} = \boldsymbol{a}(x, x) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

Advection of tracer Θ

Large scales:

$$\boldsymbol{w}$$

Small scales:

$$\sigma \dot{\boldsymbol{B}}$$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \nabla \Theta + \sigma \dot{\boldsymbol{B}} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} \boldsymbol{a} \nabla \Theta \right)$$

Large scales:

$$\boldsymbol{w}$$

Small scales:

$$\sigma \dot{\boldsymbol{B}}$$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

Advection of tracer Θ

Advection

$$\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \sigma \dot{\boldsymbol{B}} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} \boldsymbol{a} \nabla \Theta \right)$$

Large scales:

$$\boldsymbol{w}$$

Small scales:

$$\sigma \dot{\boldsymbol{B}}$$

Variance tensor:

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Advection of tracer Θ

$$\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \sigma \dot{\boldsymbol{B}} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} \boldsymbol{a} \nabla \Theta \right)$$

Advection

Diffusion

Large scales:

$$\mathbf{w}$$

Small scales:

$$\sigma \dot{\mathbf{B}}$$

Variance tensor:

$$\mathbf{a} = \mathbf{a}(\mathbf{x}, \mathbf{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\mathbf{B} (\boldsymbol{\sigma} d\mathbf{B})^T\}}{dt}$$

Advection of tracer Θ

Advection

$$\partial_t \Theta + \mathbf{w}^* \cdot \nabla \Theta + \sigma \dot{\mathbf{B}} \cdot \nabla \Theta =$$

Diffusion

$$\nabla \cdot \left(\frac{1}{2} \mathbf{a} \nabla \Theta \right)$$



Drift correction

Large scales:
 w

Small scales:
 $\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

Multiplicative
random
forcing

Advection

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta =$$

Diffusion

$$\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Drift correction

Large scales:
 w

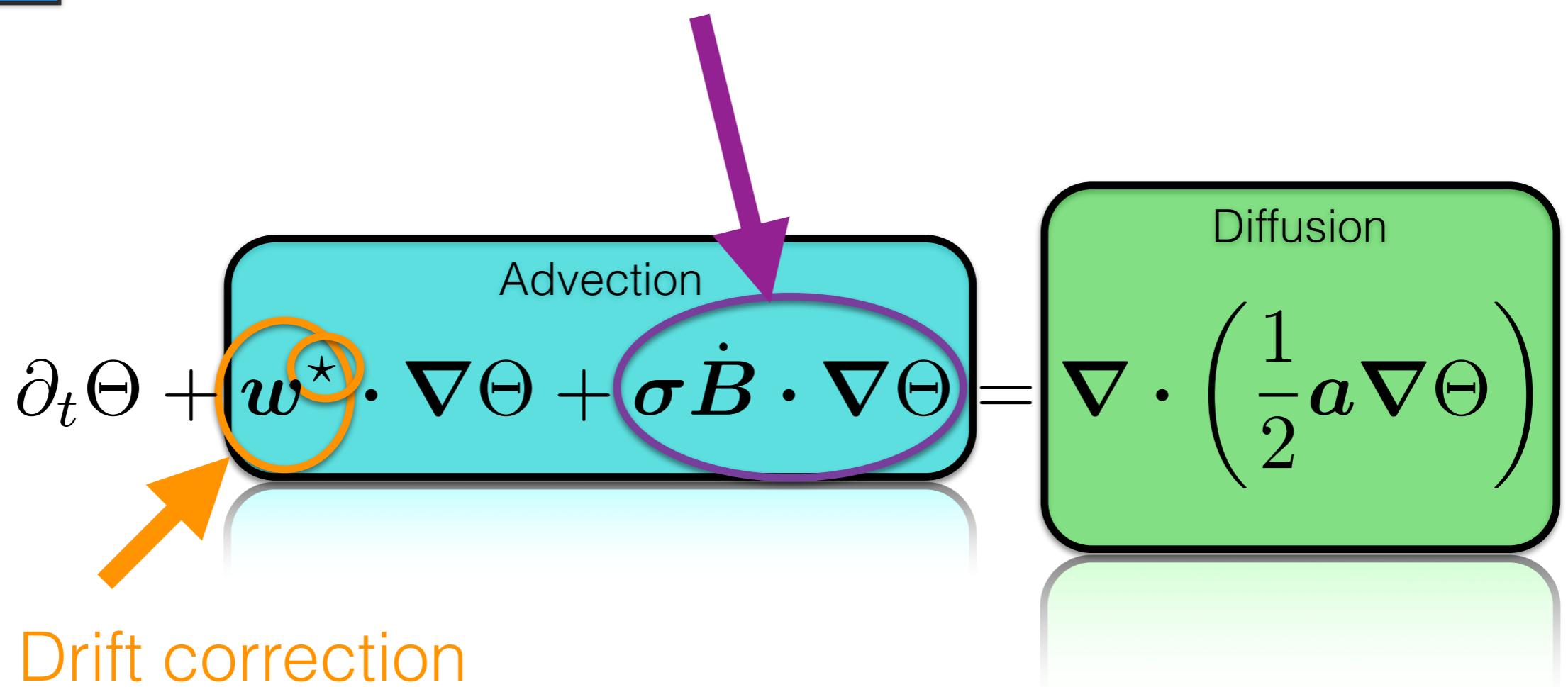
Small scales:
 $\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

Multiplicative
random
forcing



Large scales:
 w

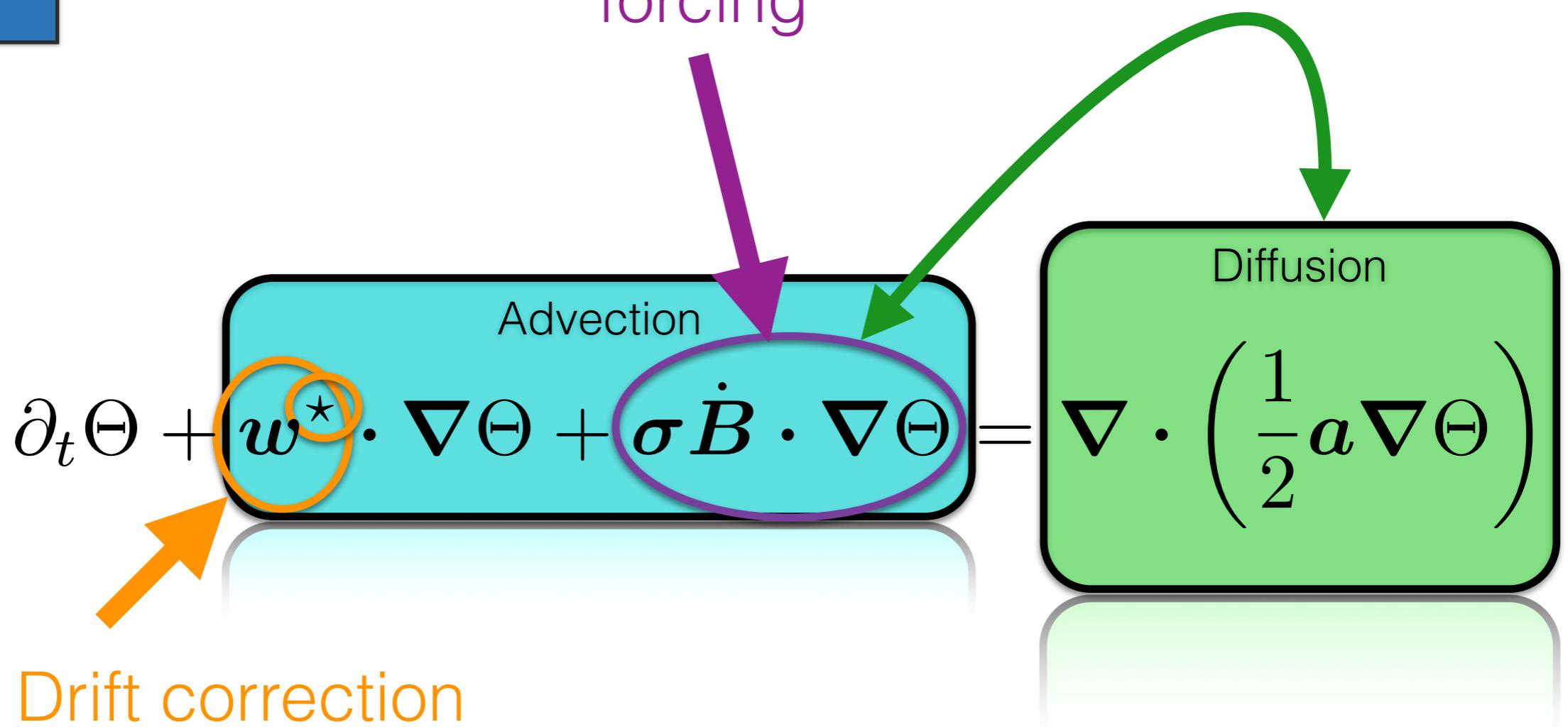
Small scales:
 $\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

Multiplicative
random
forcing



Large scales:
 w

Small scales:
 $\sigma \dot{B}$

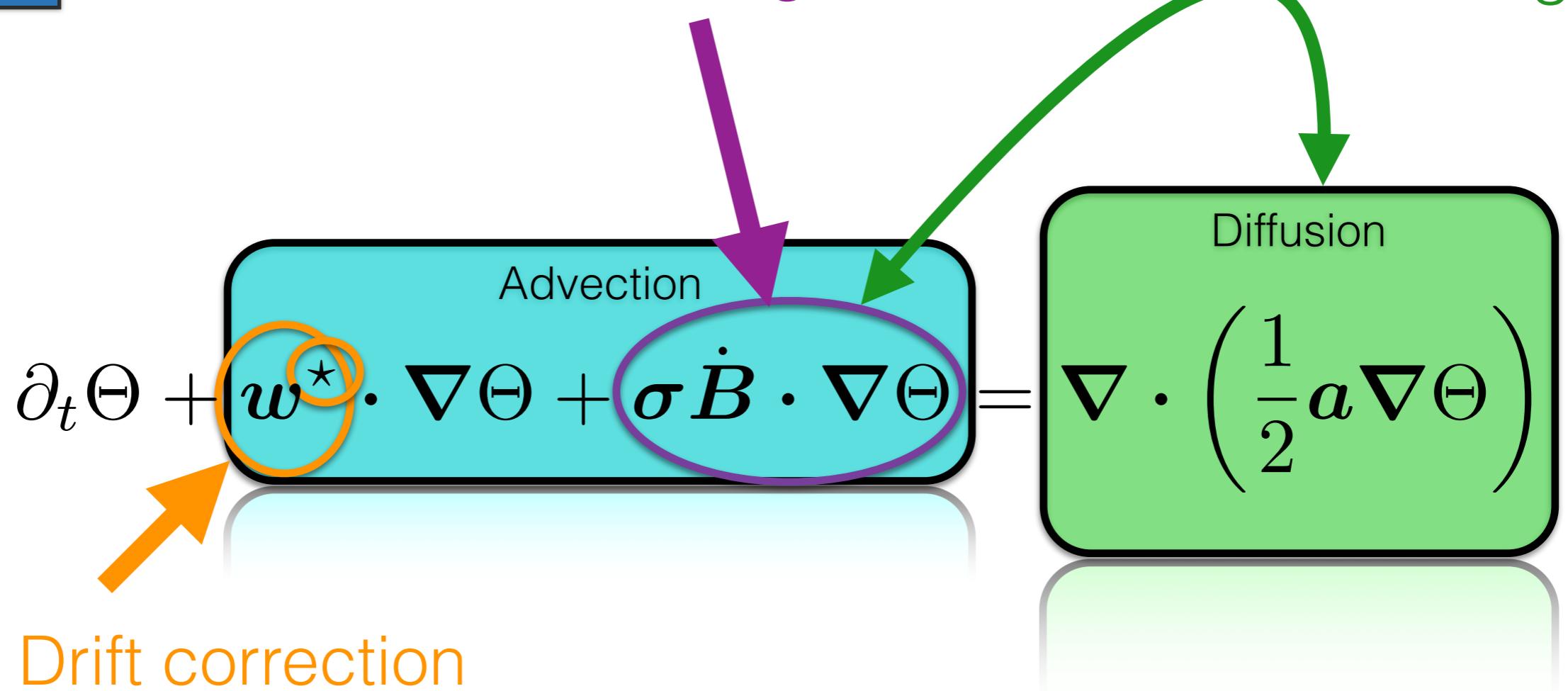
Variance
tensor:
 $a = a(x, x) =$

$$\frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

Multiplicative
random
forcing

Balanced
energy
exchanges



Part II

Some parameterization of LU models

Part II

Some parameterization of LU models

σ =? .

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance tensor:

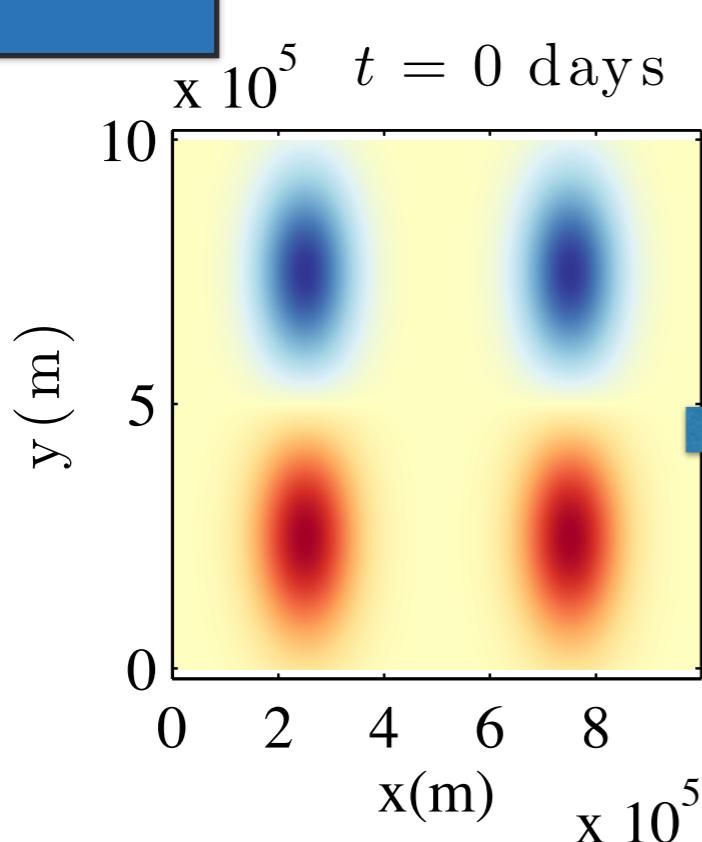
$$\mathbf{a} = \mathbf{a}(x, x) =$$

$$\frac{\mathbb{E}\{\boldsymbol{\sigma} dB (\boldsymbol{\sigma} dB)^T\}}{dt}$$

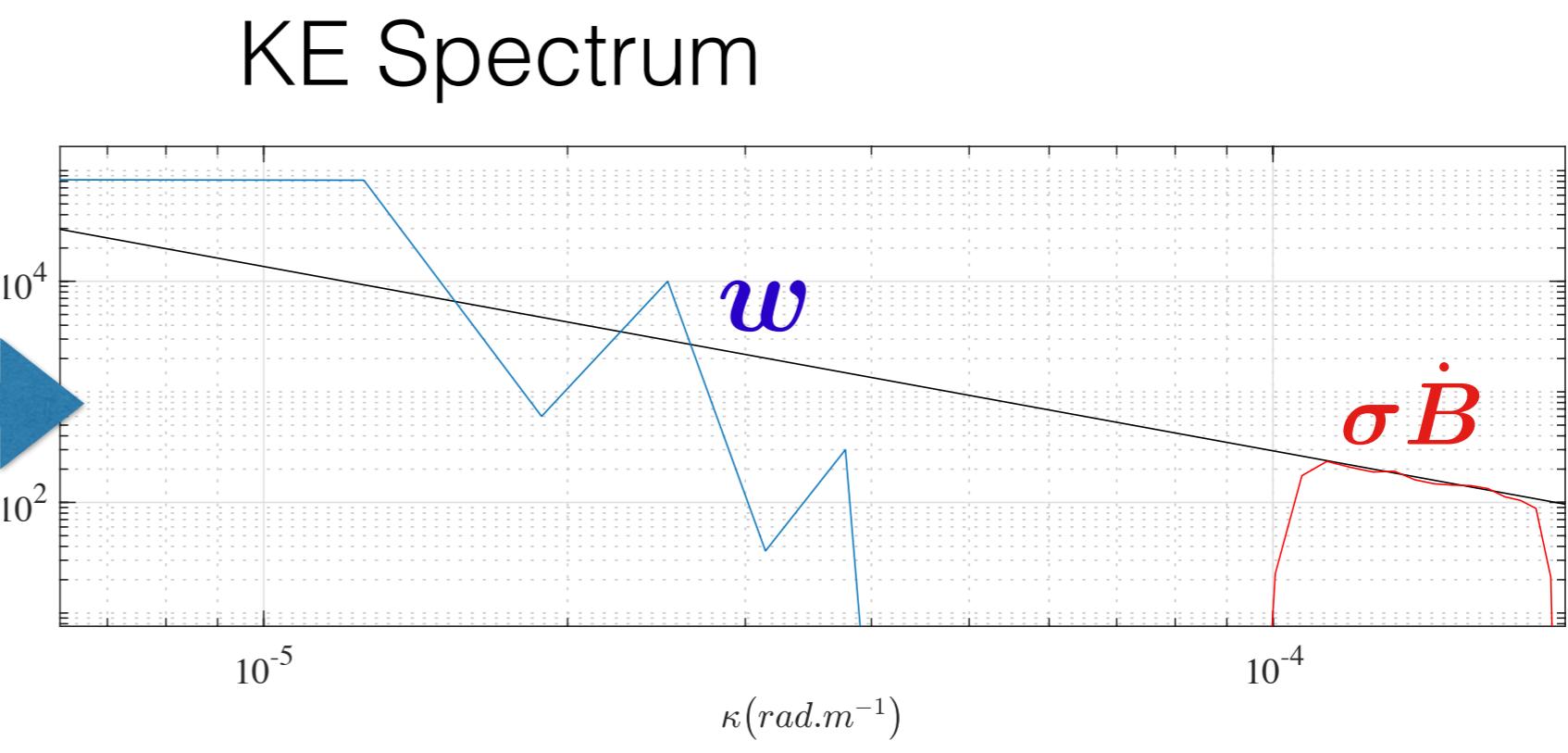
MU Spec

Spectral model

(homogeneous and stationary $\sigma \dot{B}$)



KE Spectrum



Code online

Reference:

Resseguier, Memin & Chapron 2017b

Large scales:

$$w$$

Small scales:

$$\sigma \dot{B}$$

Variance tensor:

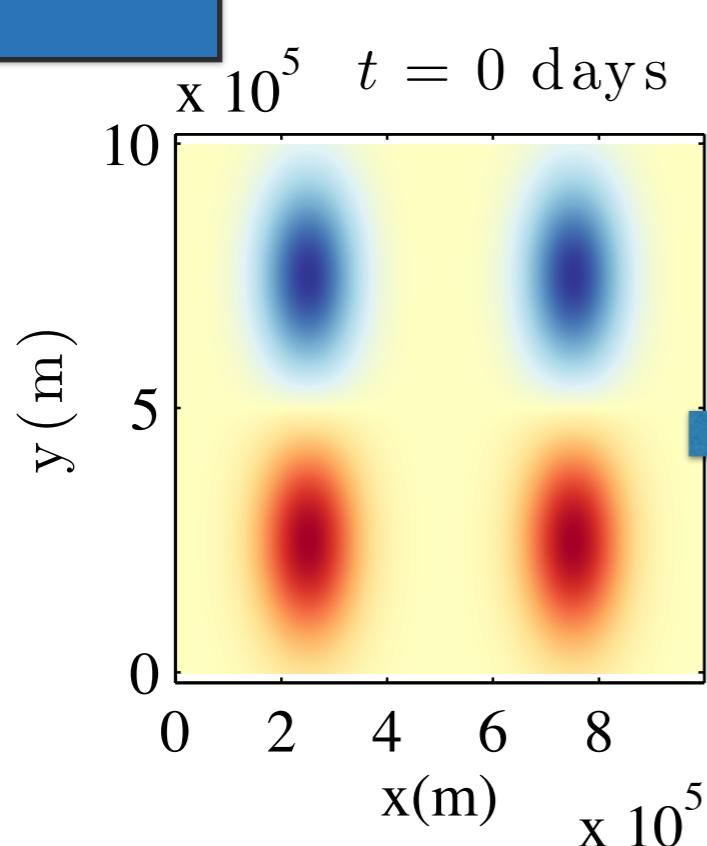
$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

MU Spec

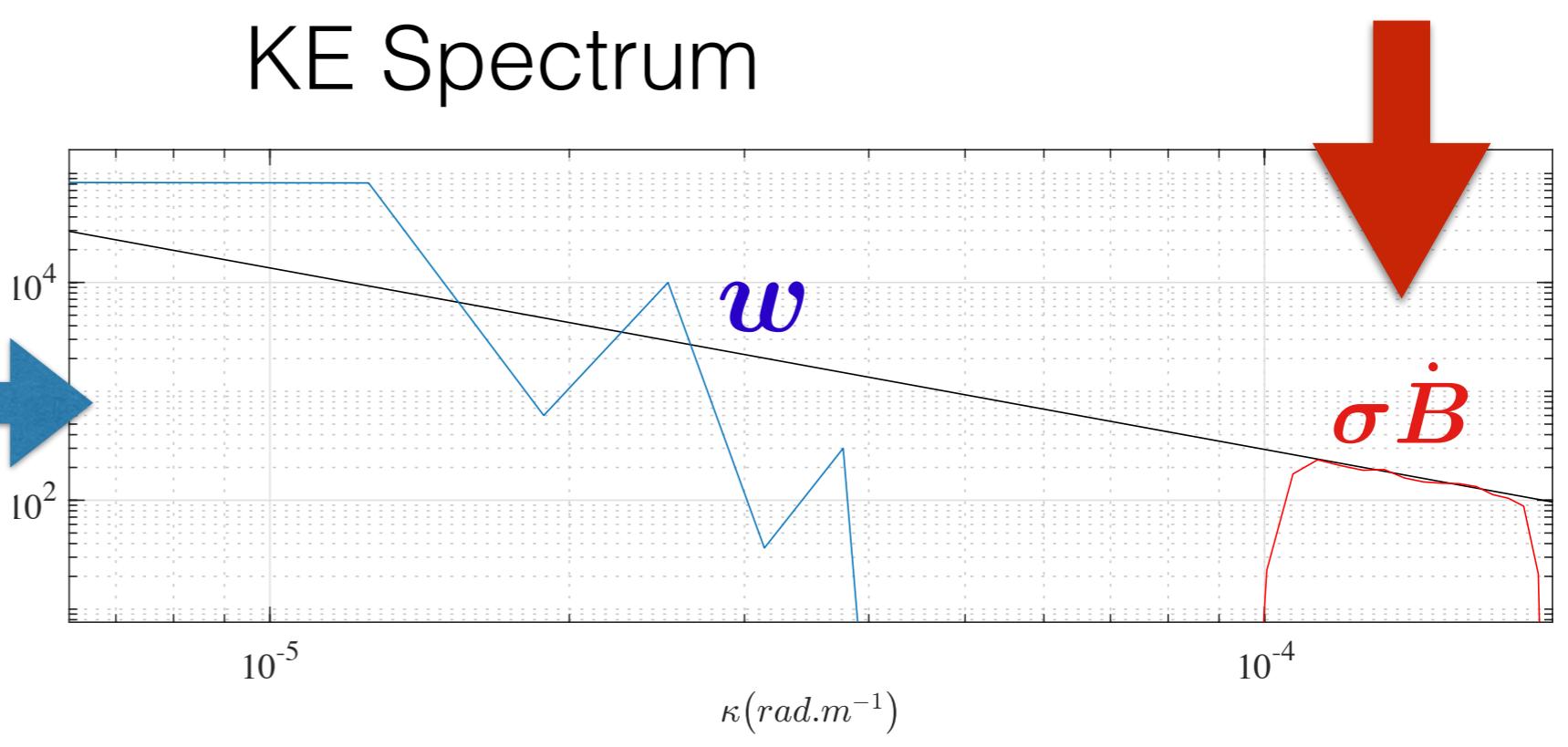
Spectral model

(homogeneous and stationary $\sigma \dot{B}$)

Fixed spectrum at small scales



KE Spectrum



Code online

Large scales:

$$w$$

Small scales:

$$\sigma \dot{B}$$

Variance tensor:

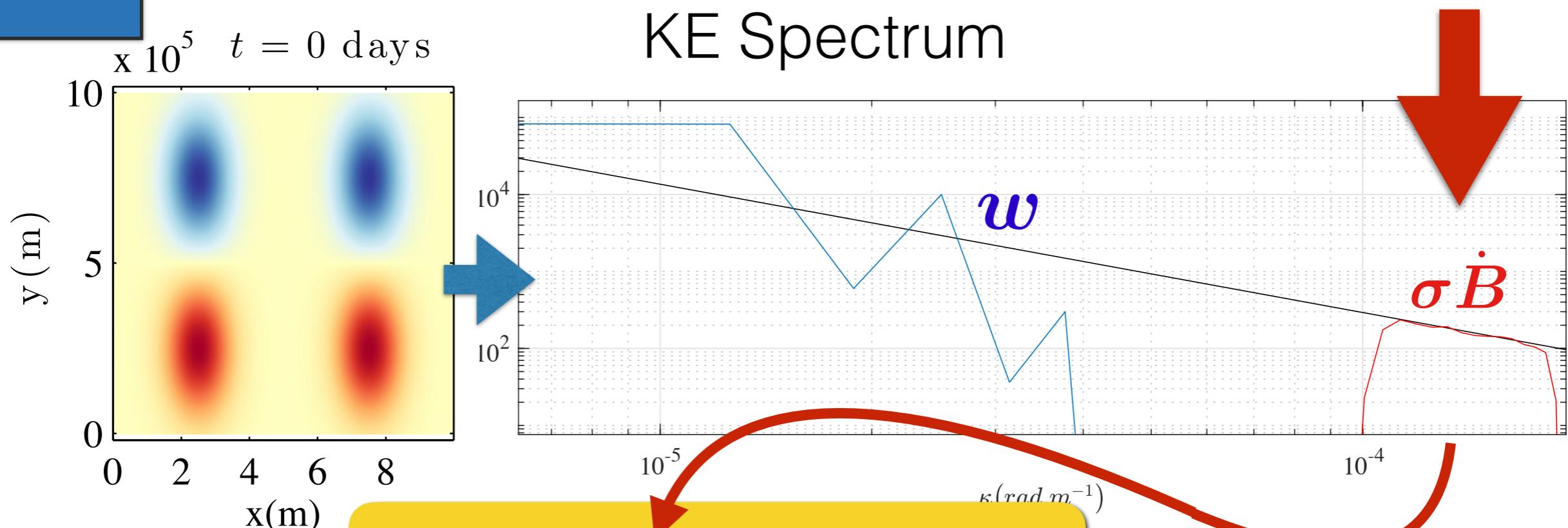
$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

MU Spec

Spectral model

(homogeneous and stationary $\sigma \dot{B}$)

Fixed spectrum at small scales



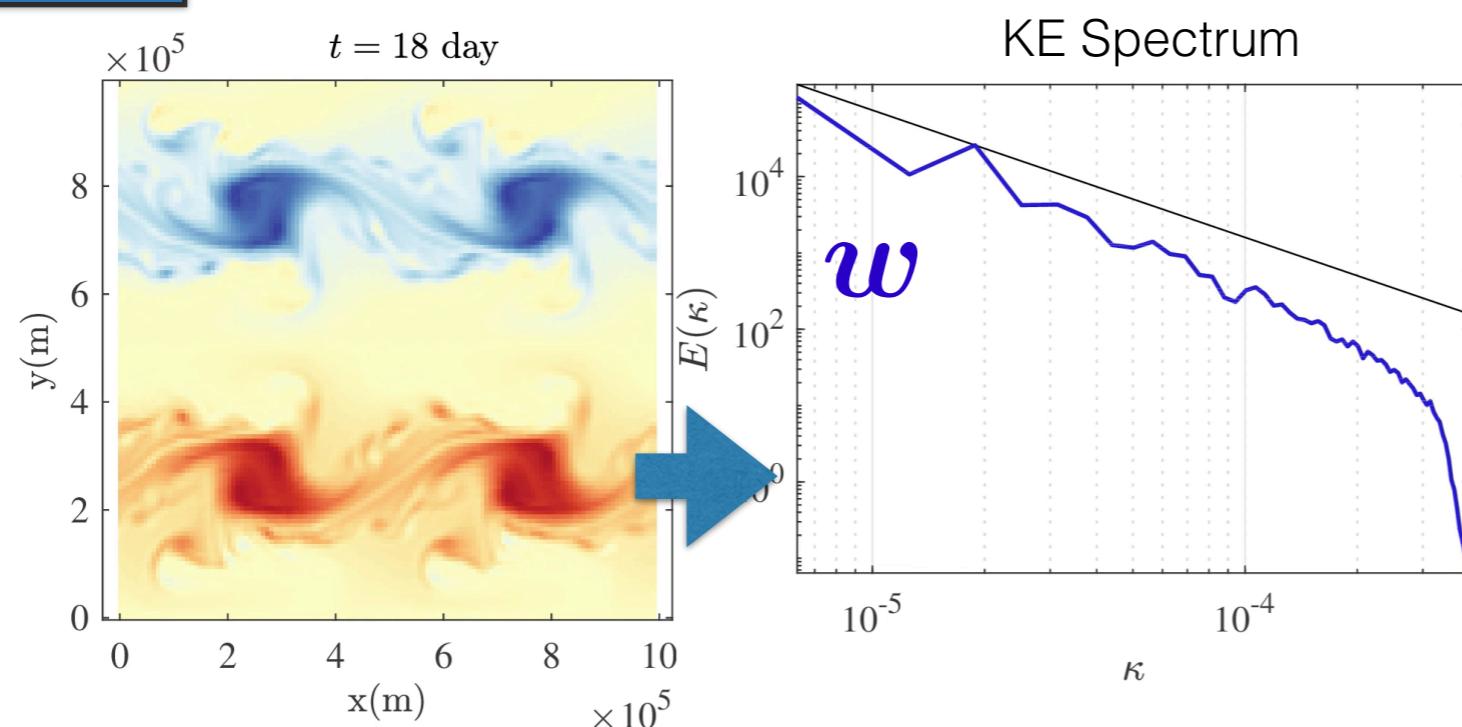
Code online

Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

Absolute Diffusivity Spectral Density

MU ADSD

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)



Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

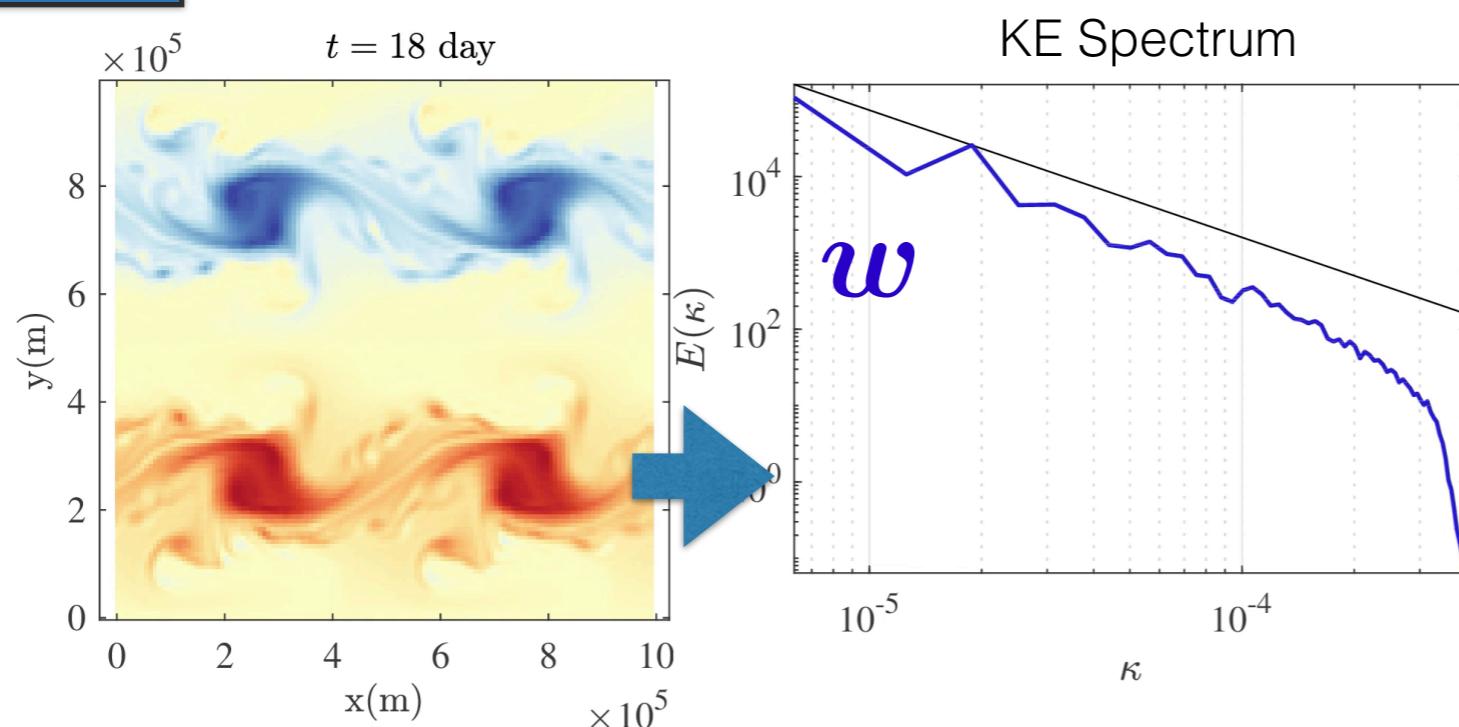
Absolute Diffusivity Spectral Density

MU ADSD

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Absolute Diffusivity
Spectral Density

$$A(\kappa) = E(\kappa)\tau(\kappa)$$



Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance
 tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

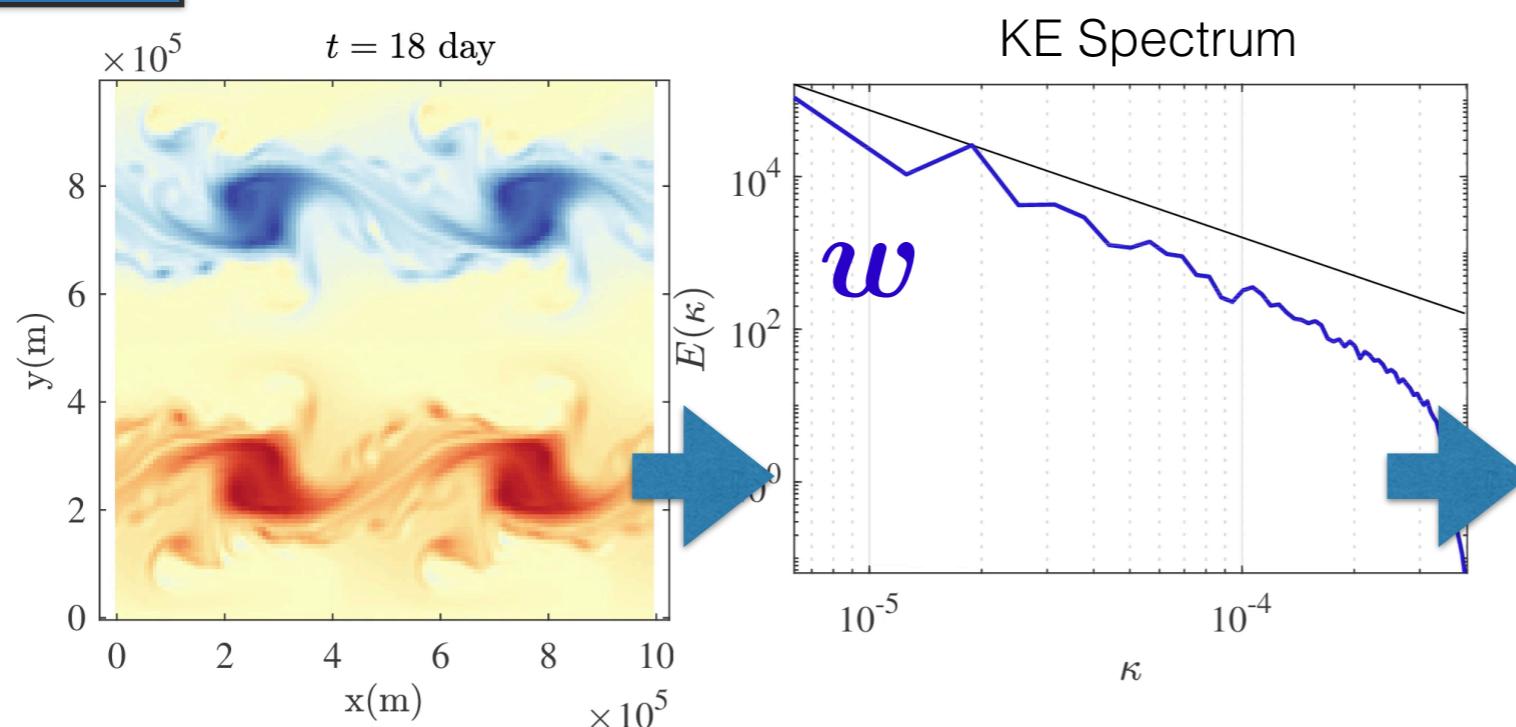
Absolute Diffusivity Spectral Density

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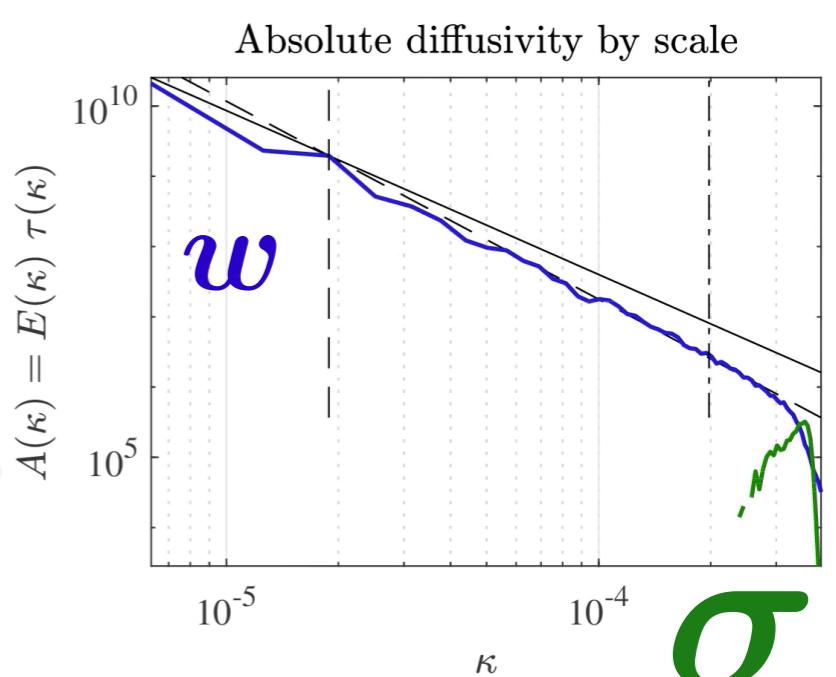
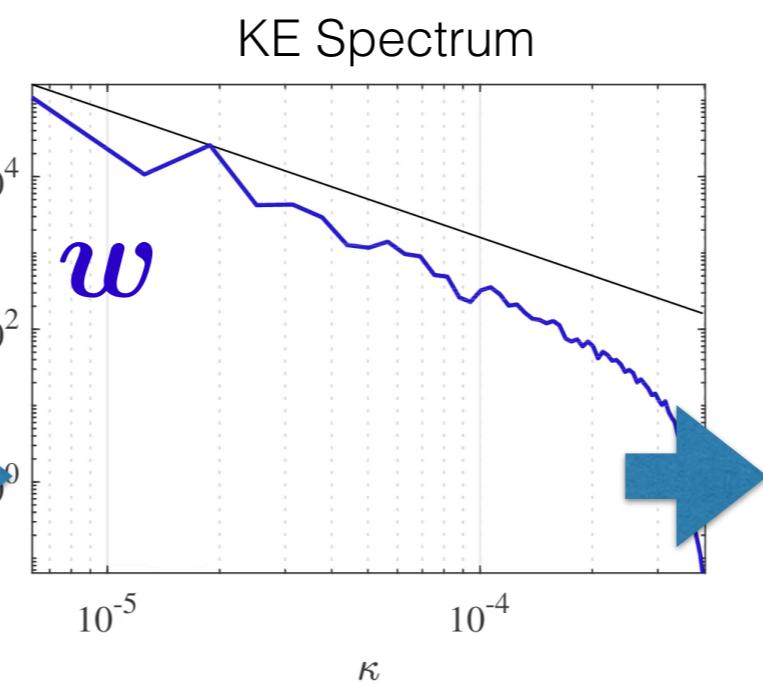
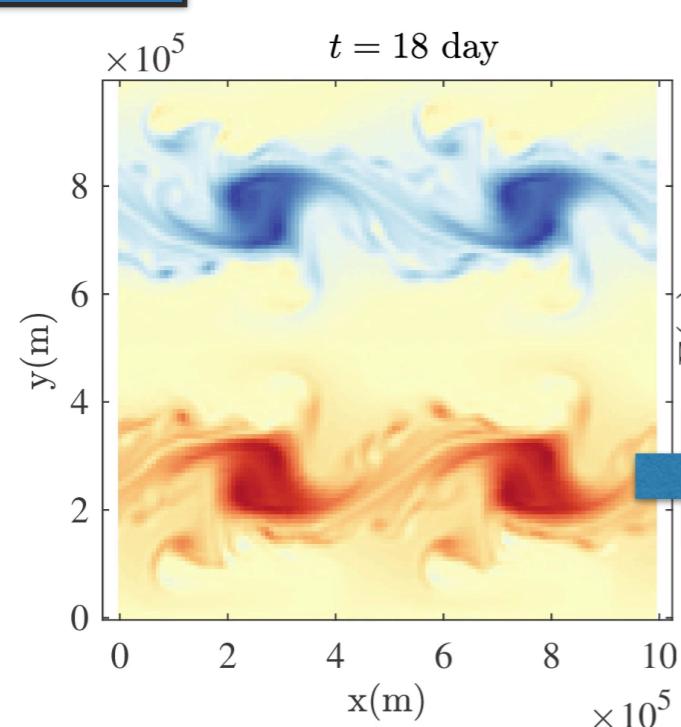
Absolute Diffusivity Spectral Density

MU ADSD

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Absolute Diffusivity
Spectral Density

$$A(\kappa) = E(\kappa)\tau(\kappa)$$



Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance
 tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

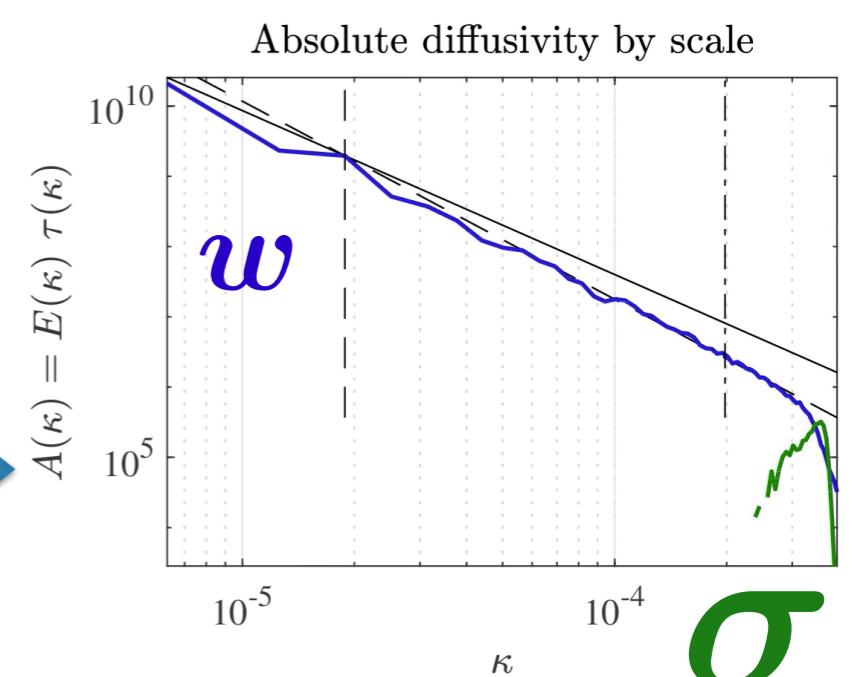
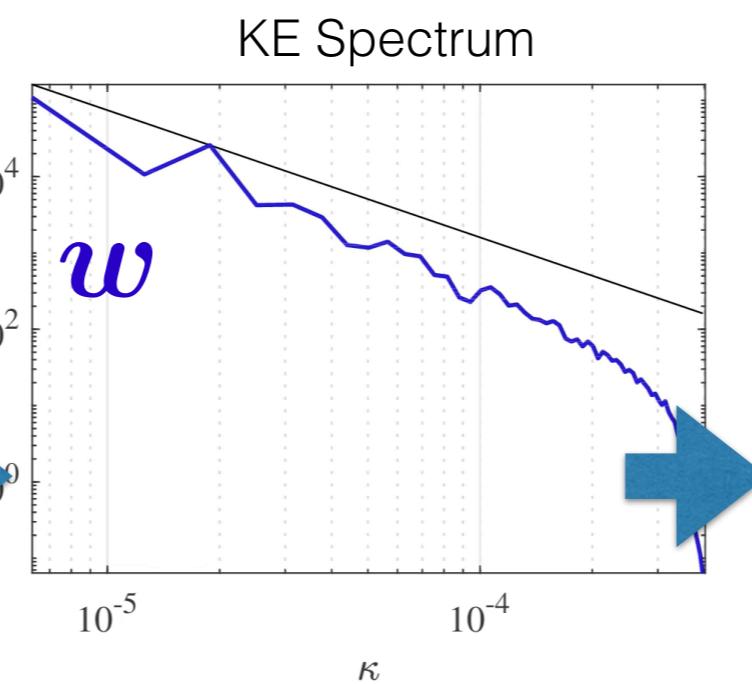
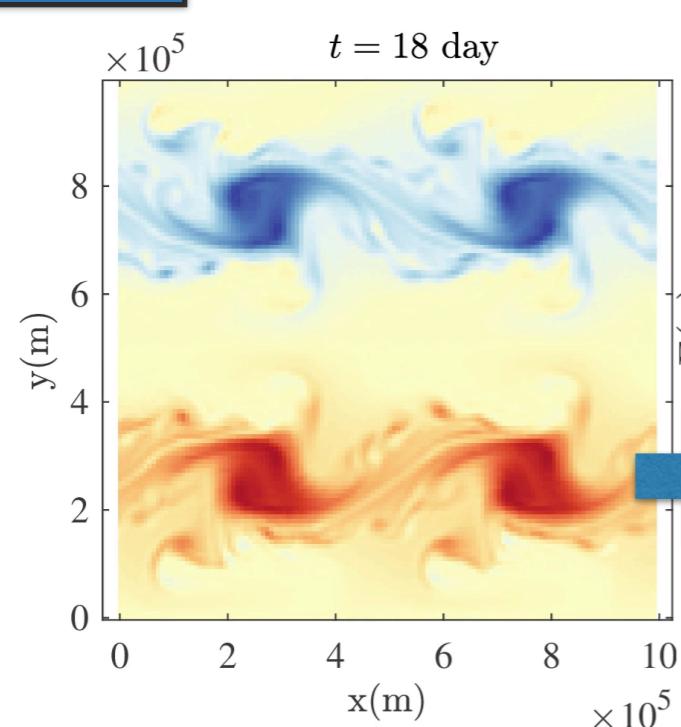
Absolute Diffusivity Spectral Density

MU ADSD

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Absolute Diffusivity
Spectral Density

$$A(\kappa) = E(\kappa)\tau(\kappa)$$



Large scales:
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 Small scales:
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 Variance
 tensor:
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Absolute Diffusivity Spectral Density

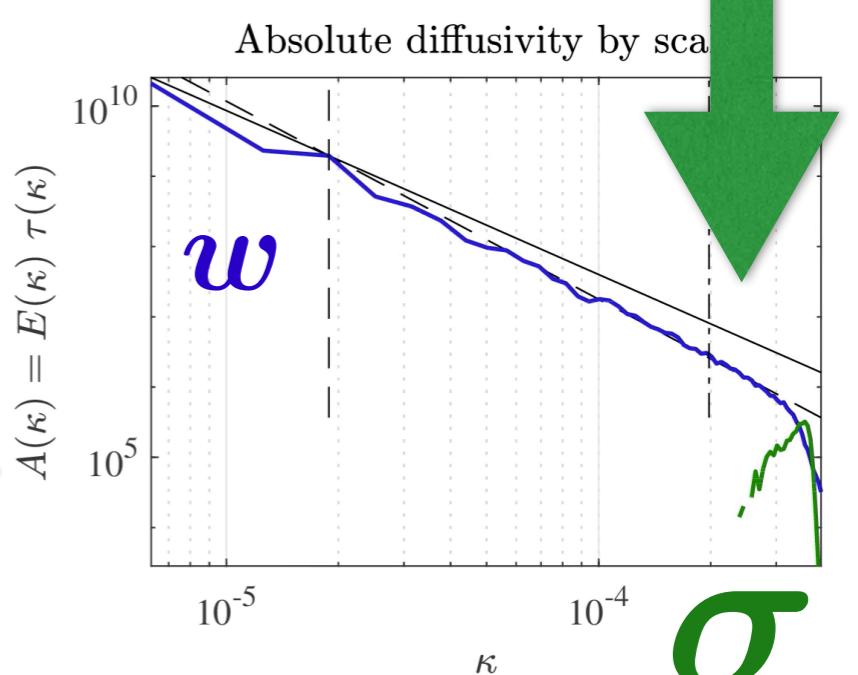
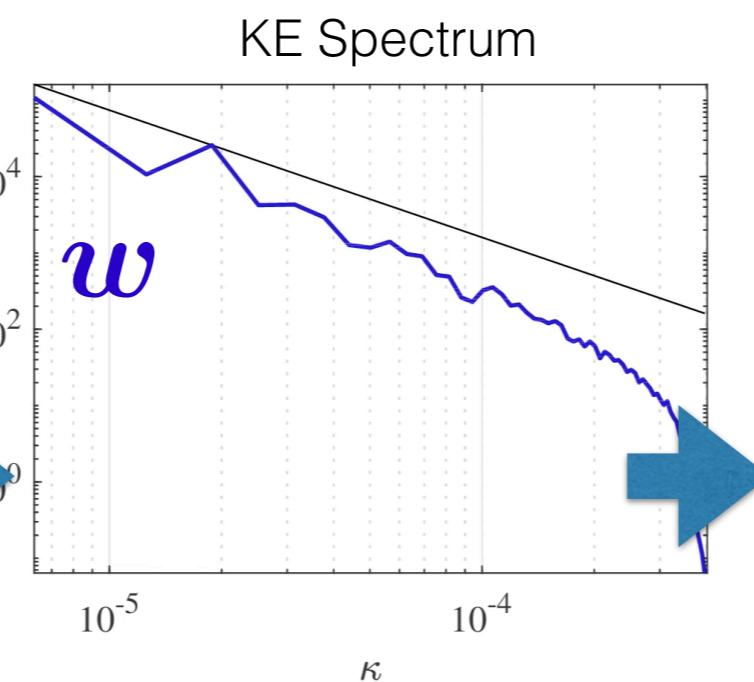
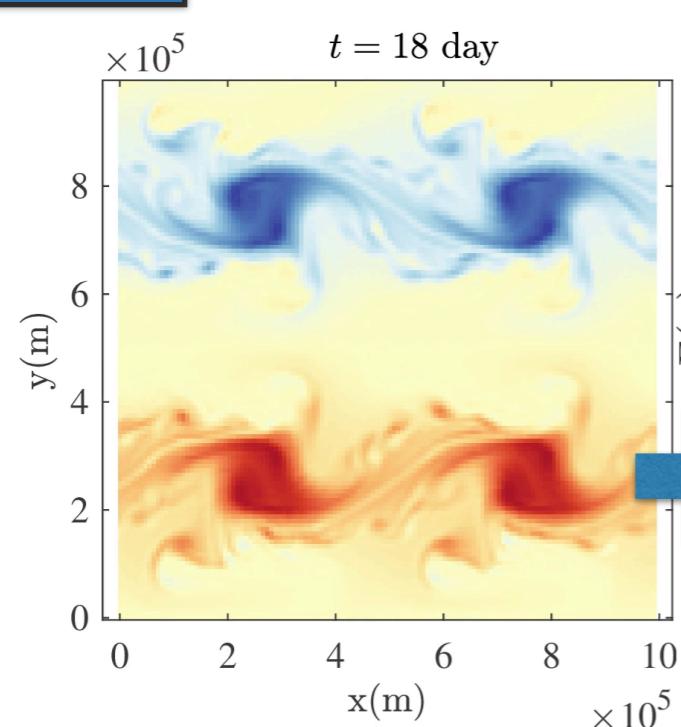
MU ADSD

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Absolute Diffusivity
Spectral Density

$$A(\kappa) = E(\kappa)\tau(\kappa)$$

Residual
non-stationary
ADSD



Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance
 tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

Absolute Diffusivity Spectral Density

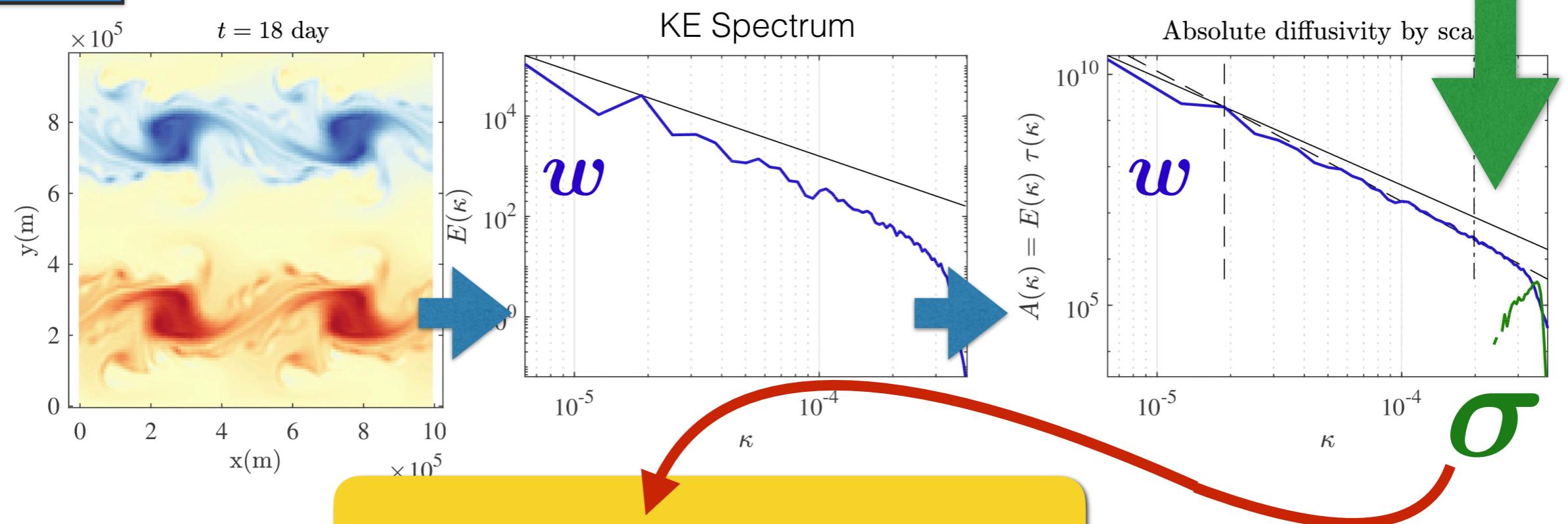
MU ADSD

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Absolute Diffusivity
Spectral Density

$$A(\kappa) = E(\kappa)\tau(\kappa)$$

Residual
non-stationary
ADSD



Large scales:

w

Small scales:

$\sigma \dot{B}$

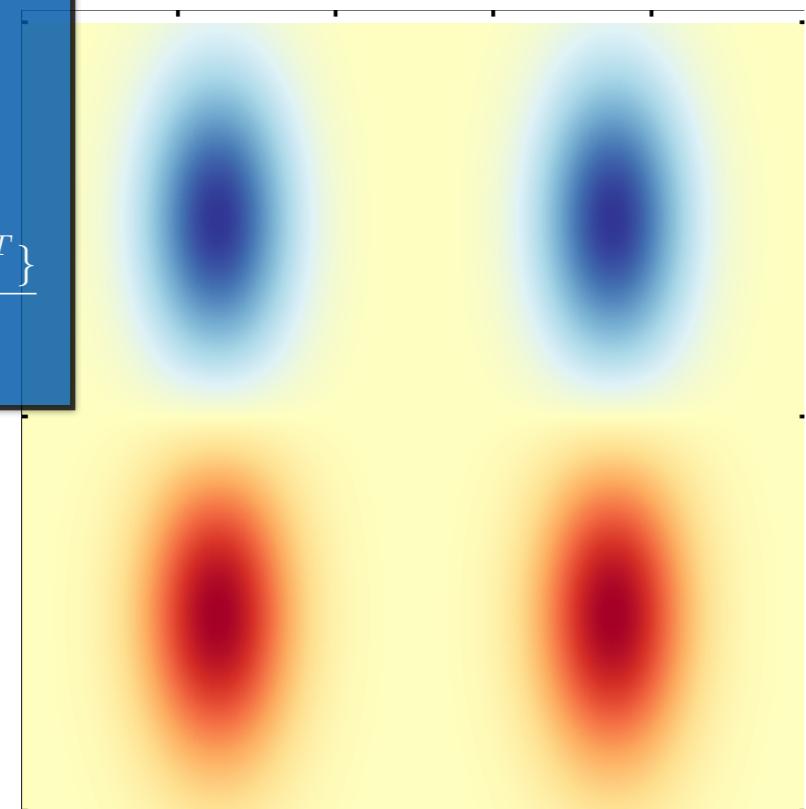
Variance
tensor:

$$\mathbf{a} = \mathbf{a}(x, x) = \frac{\mathbb{E}\{\boldsymbol{\sigma} dB (\boldsymbol{\sigma} dB)^T\}}{dt}$$

Random switching of points

MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)



Large scales:

w

Small scales:

$\sigma \dot{B}$

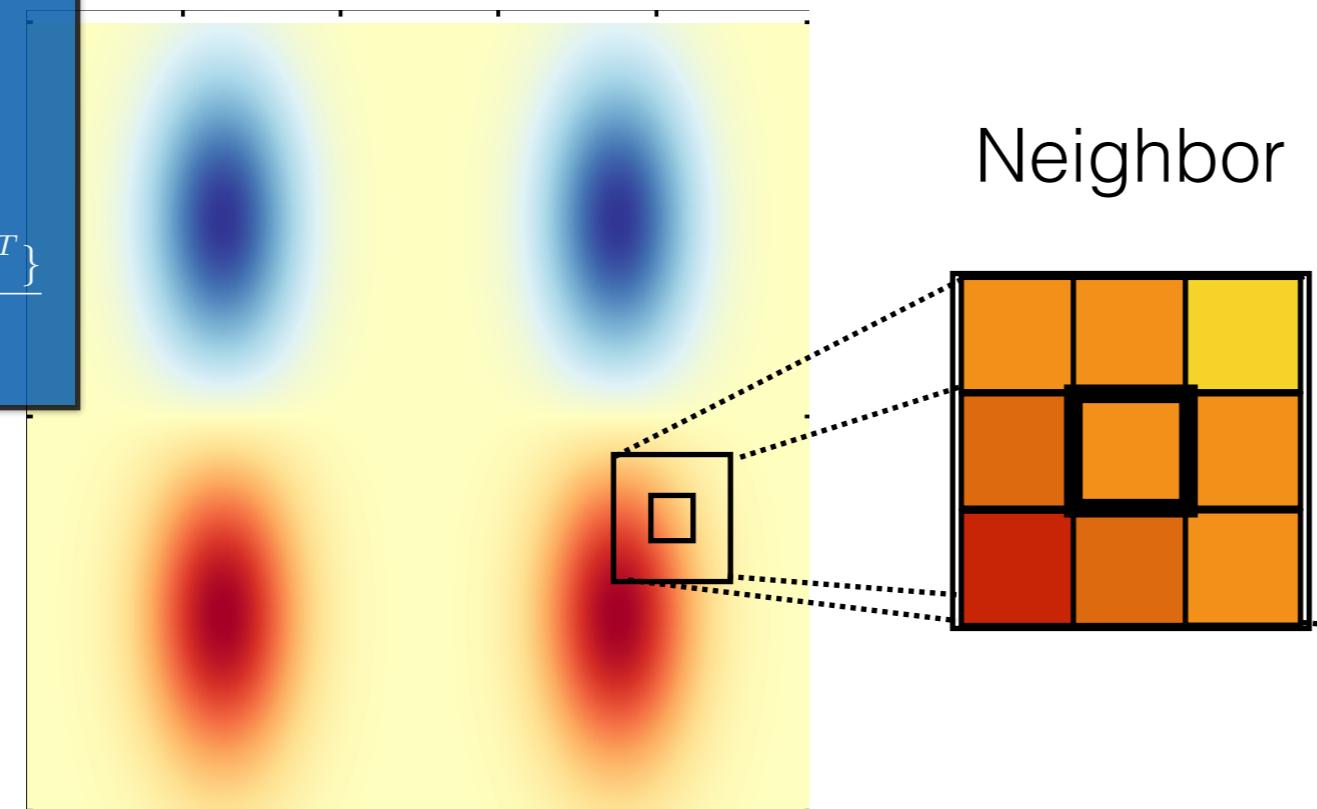
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Random switching of points

MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)



Large scales:

w

Small scales:

$\sigma \dot{B}$

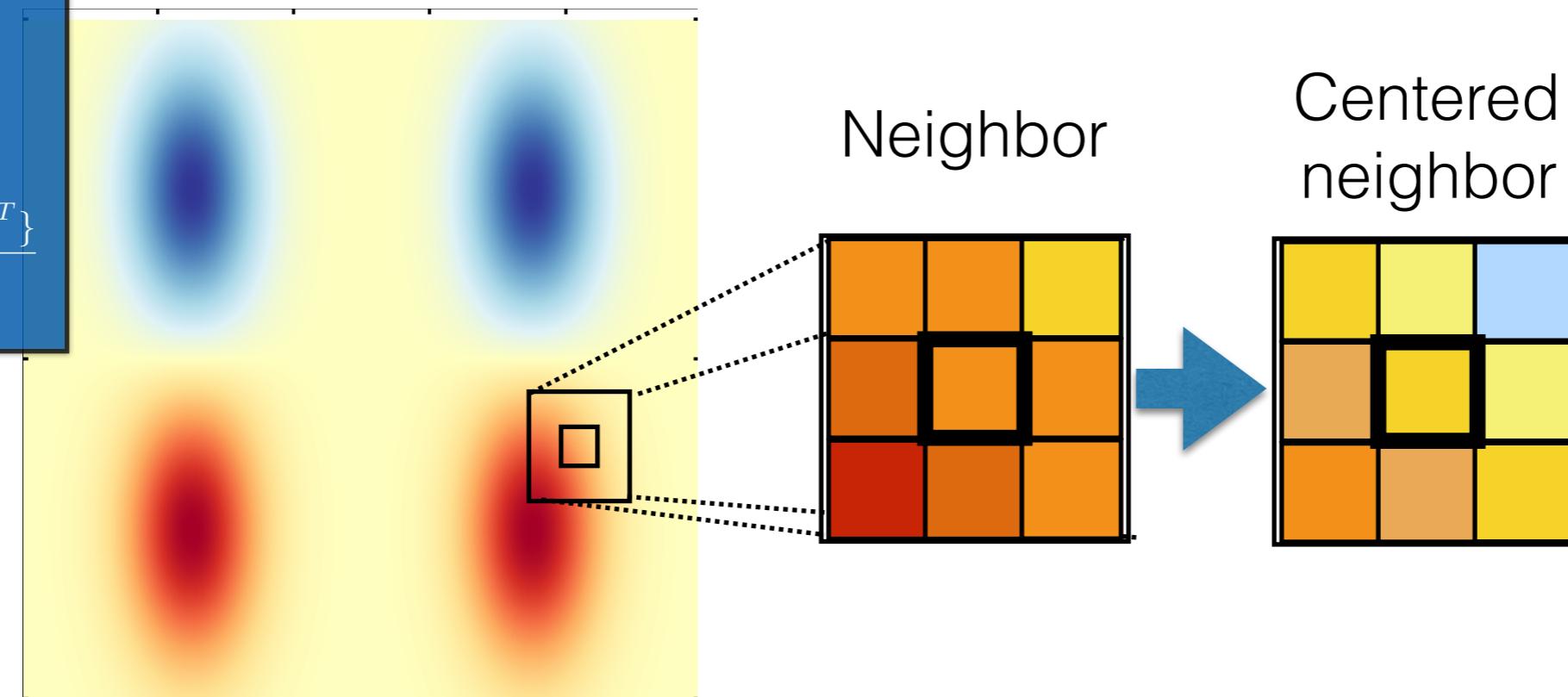
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Random switching of points

MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)



Large scales:

w

Small scales:

$\sigma \dot{B}$

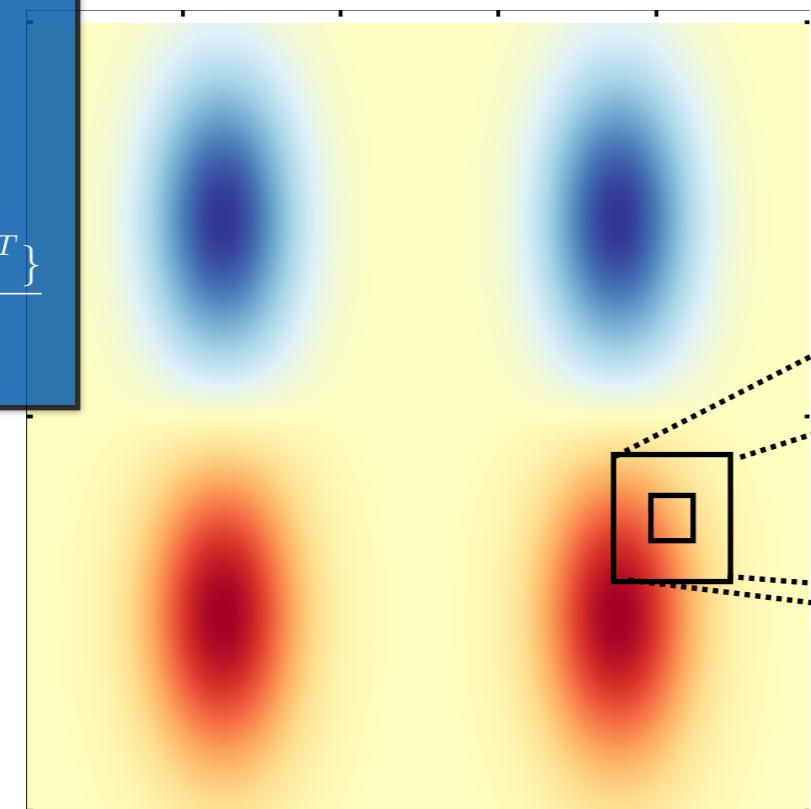
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

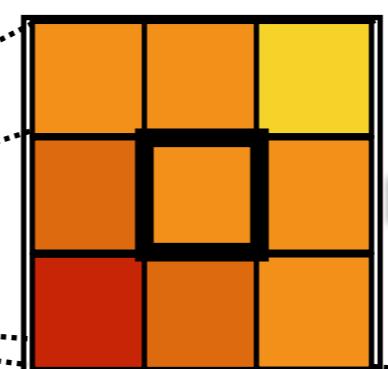
Random switching of points

MU SVD

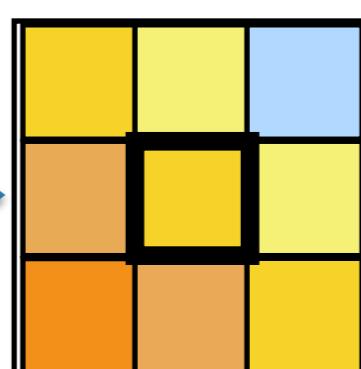
(heterogeneous and non-stationary $\sigma \dot{B}$)



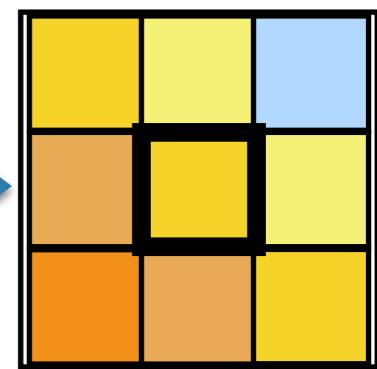
Neighbor



Centered
neighbor



Random
selection



Large scales:

w

Small scales:

$\sigma \dot{B}$

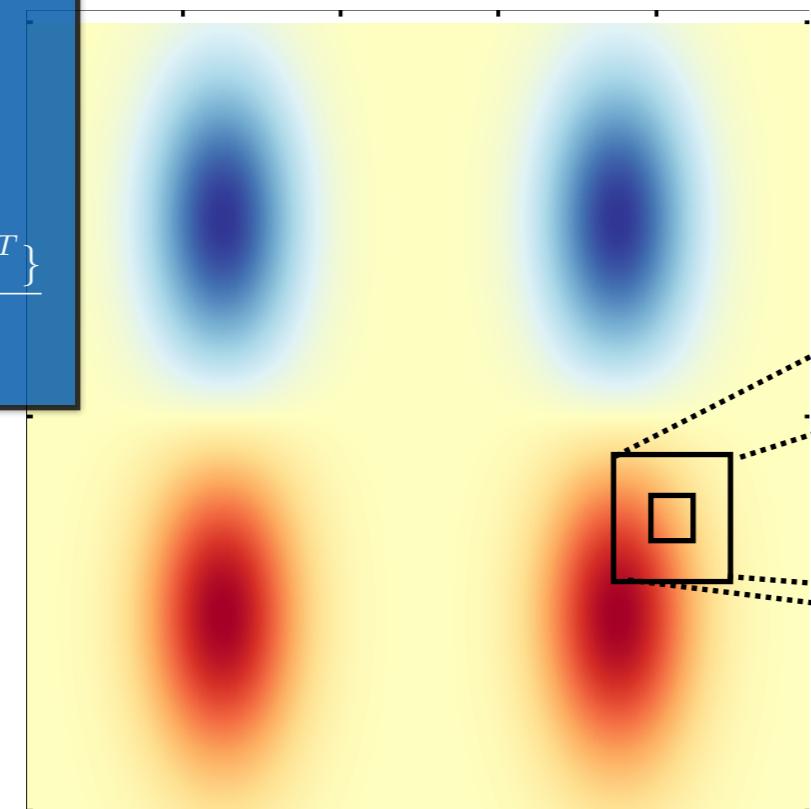
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

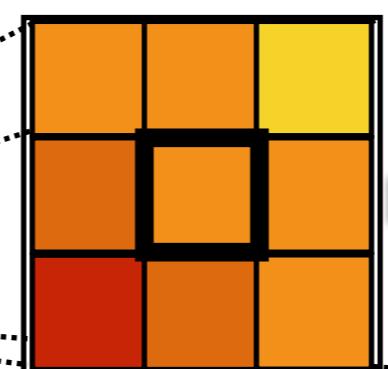
Random switching of points

MU SVD

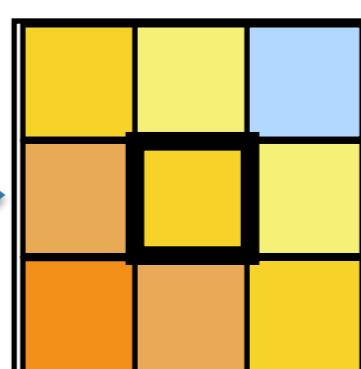
(heterogeneous and non-stationary $\sigma \dot{B}$)



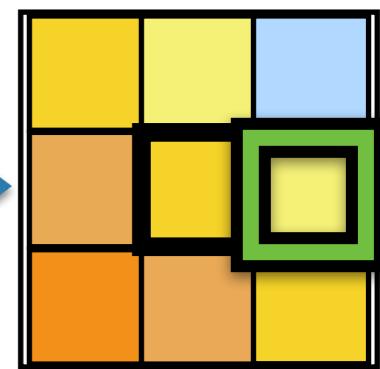
Neighbor



Centered
neighbor



Random
selection



Large scales:

w

Small scales:

$\sigma \dot{B}$

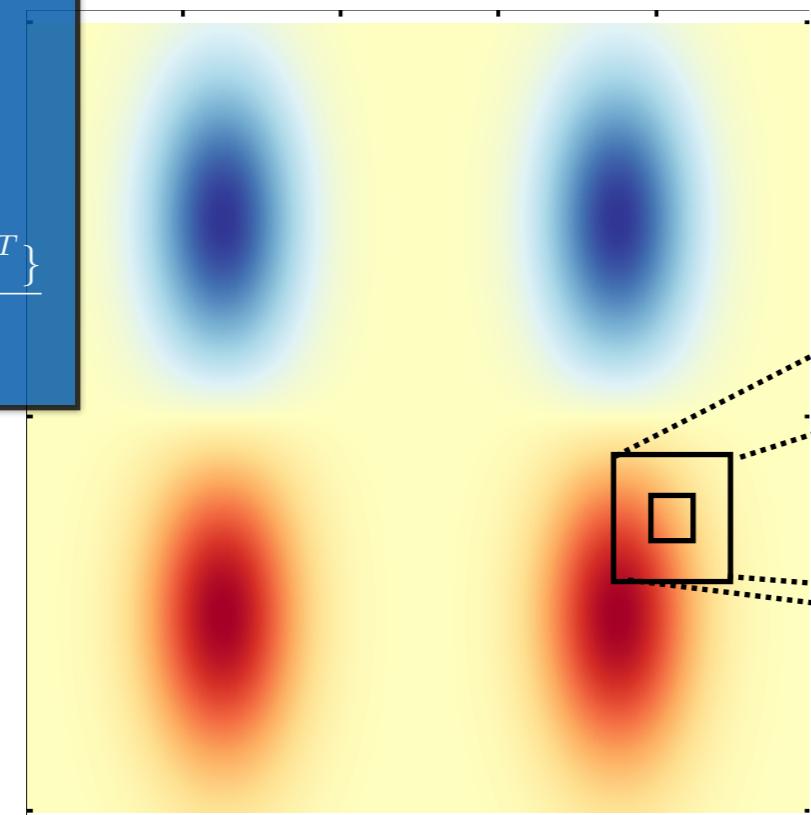
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

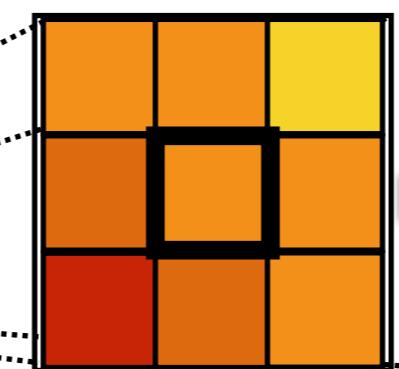
Random switching of points

MU SVD

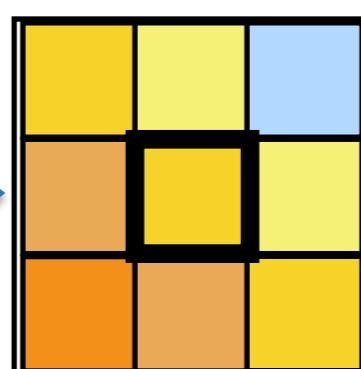
(heterogeneous and non-stationary $\sigma \dot{B}$)



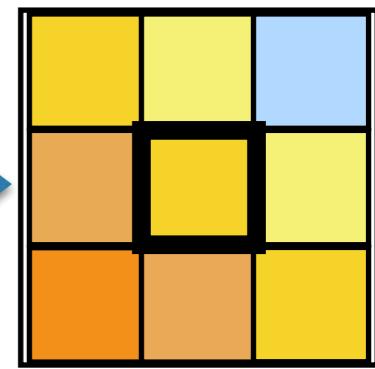
Neighbor



Centered
neighbor



Random
selection



Large scales:

w

Small scales:

$\sigma \dot{B}$

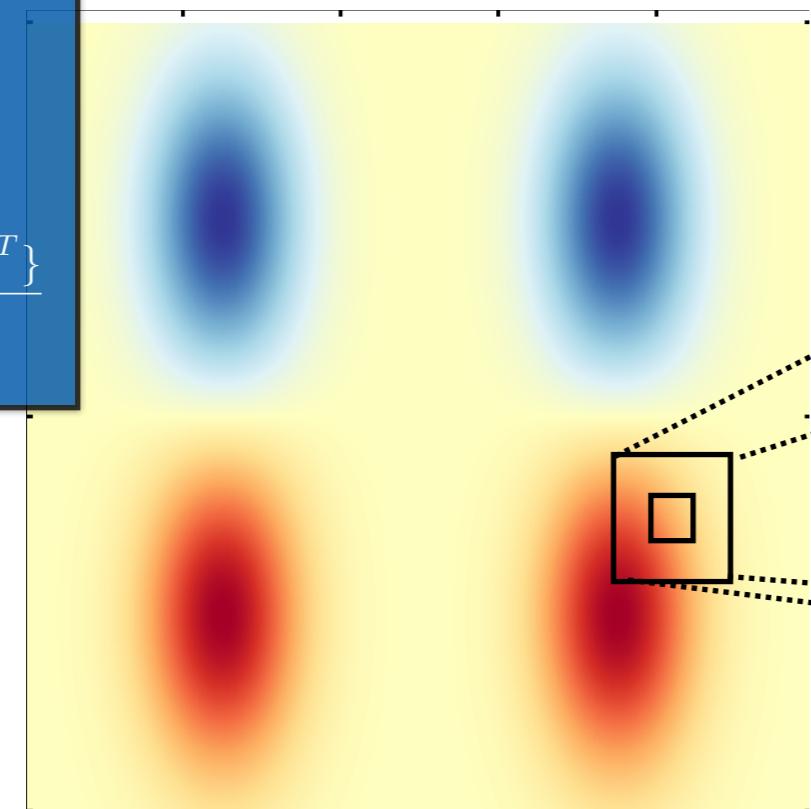
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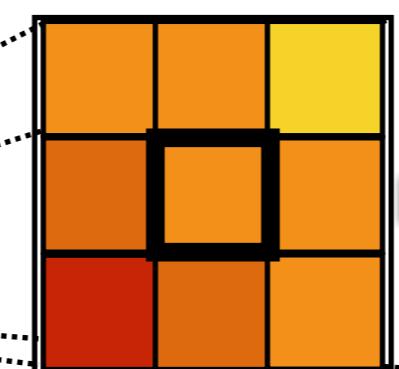
Random switching of points

MU SVD

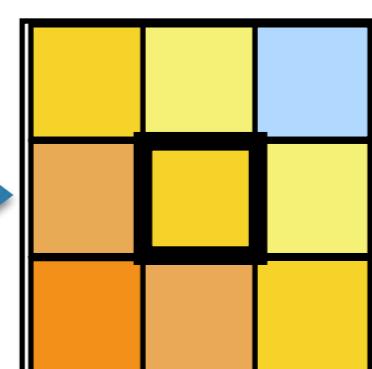
(heterogeneous and non-stationary $\sigma \dot{B}$)



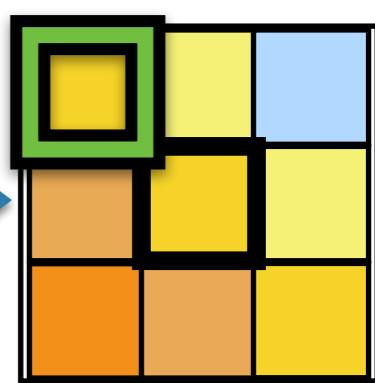
Neighbor



Centered
neighbor



Random
selection



Large scales:

w

Small scales:

$\sigma \dot{B}$

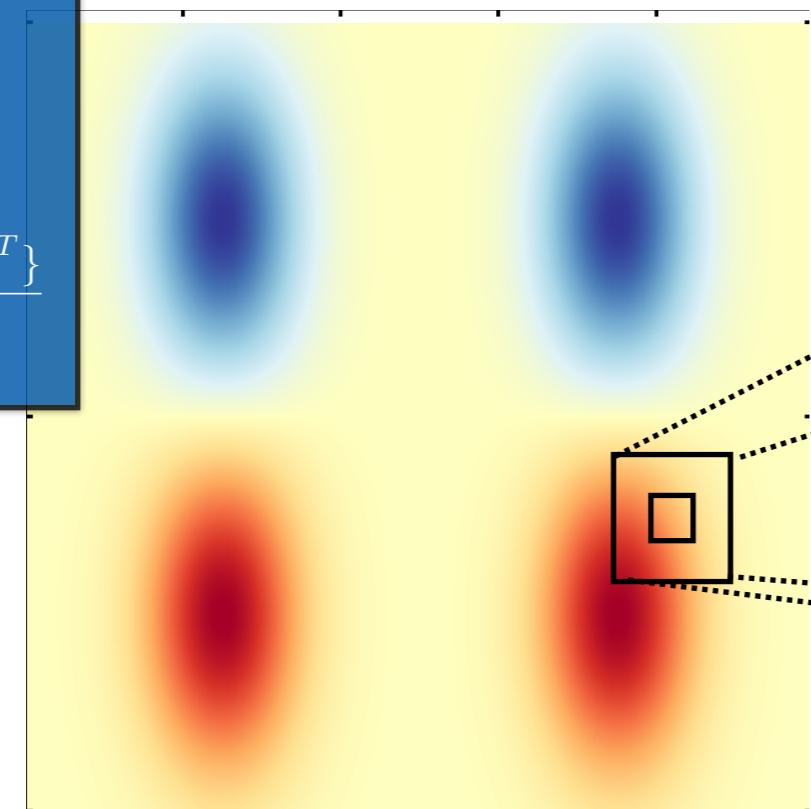
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

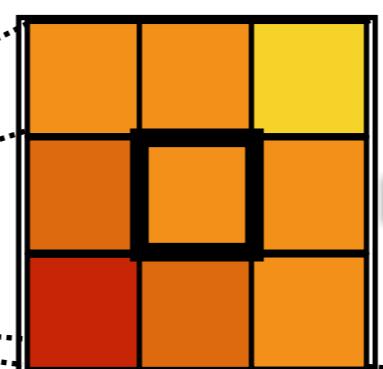
Random switching of points

MU SVD

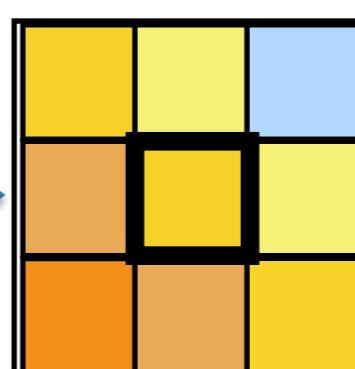
(heterogeneous and non-stationary $\sigma \dot{B}$)



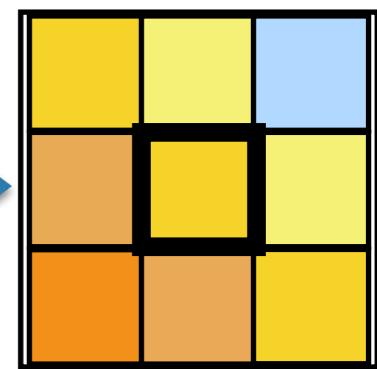
Neighbor



Centered
neighbor



Random
selection



Large scales:

w

Small scales:

$\sigma \dot{B}$

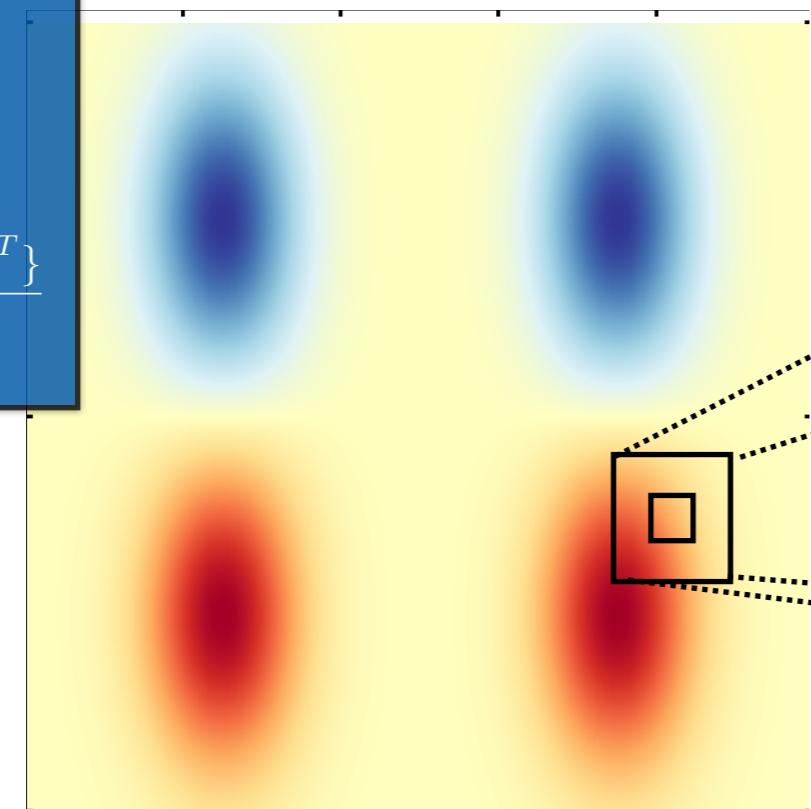
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

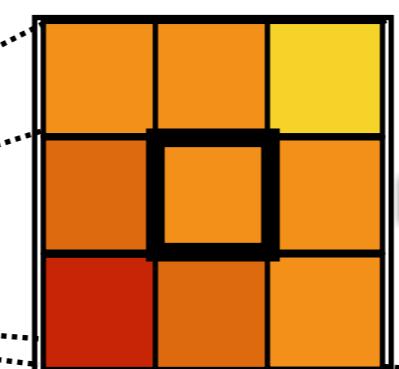
Random switching of points

MU SVD

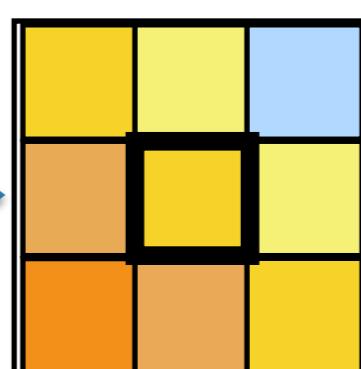
(heterogeneous and non-stationary $\sigma \dot{B}$)



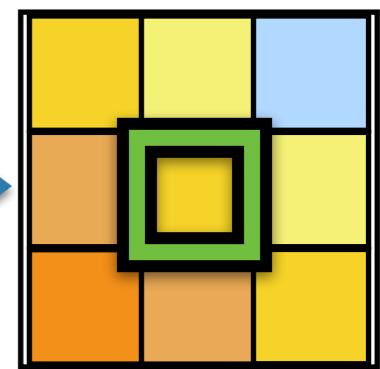
Neighbor



Centered
neighbor



Random
selection



Large scales:

w

Small scales:

$\sigma \dot{B}$

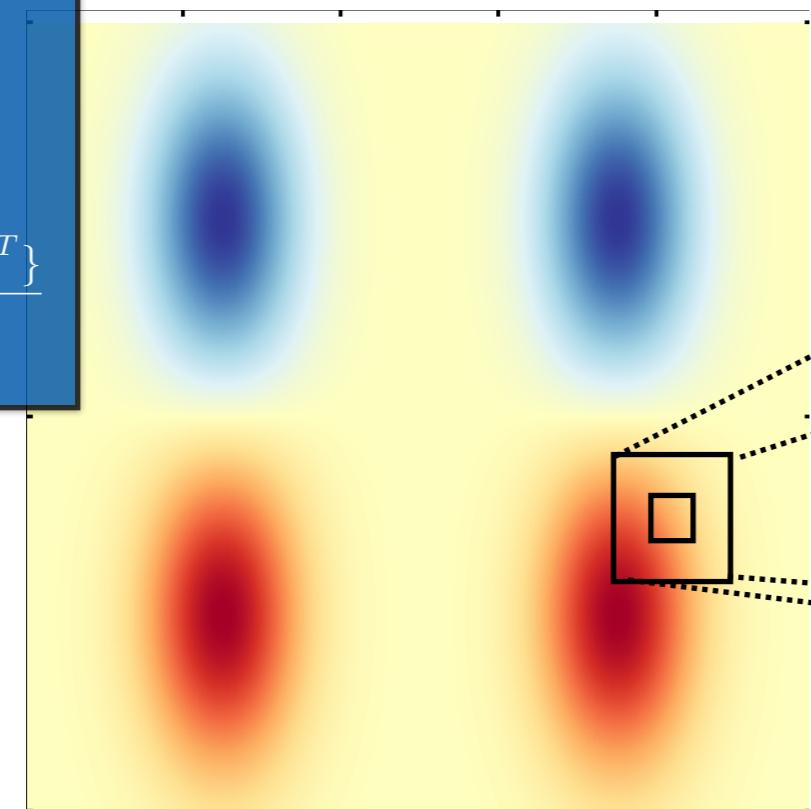
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

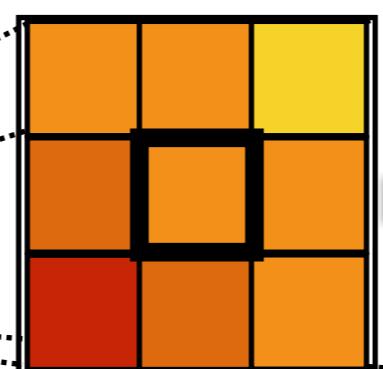
Random switching of points

MU SVD

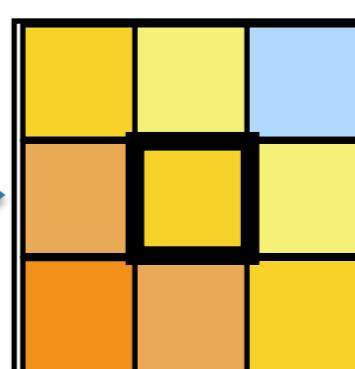
(heterogeneous and non-stationary $\sigma \dot{B}$)



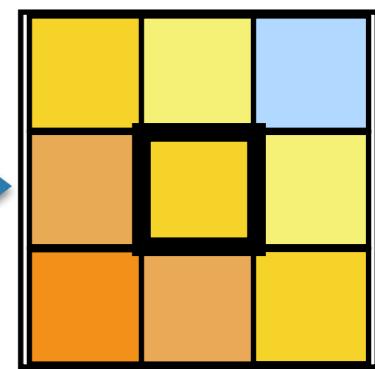
Neighbor



Centered
neighbor



Random
selection



Large scales:

w

Small scales:

$\sigma \dot{B}$

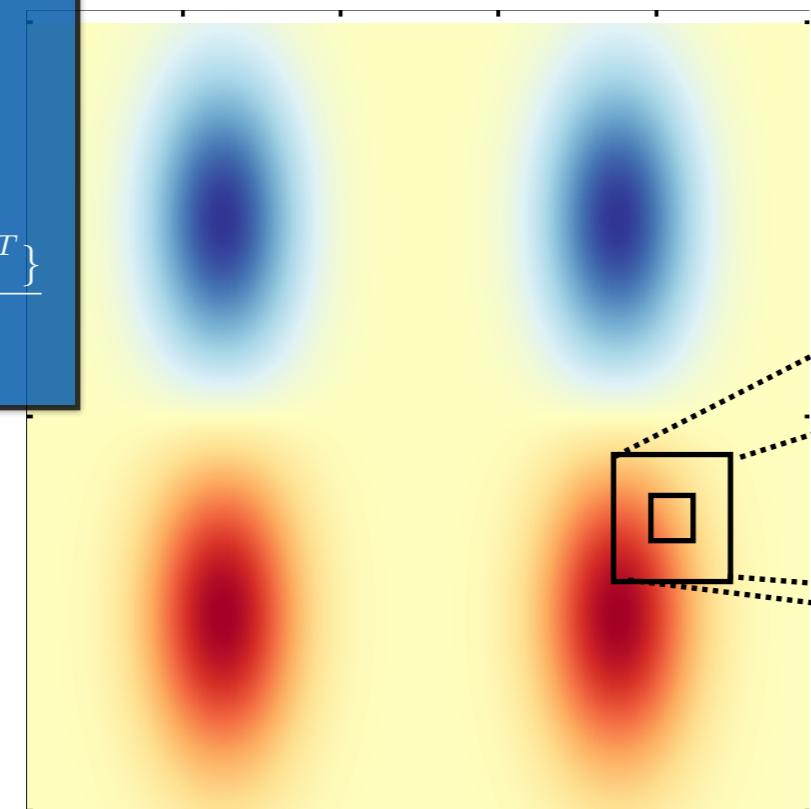
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

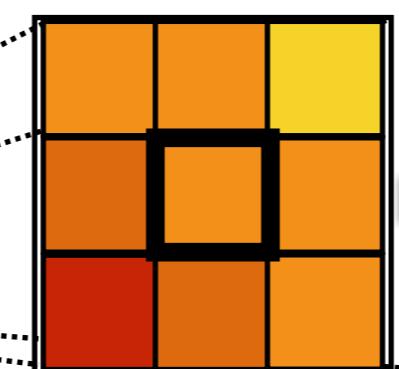
Random switching of points

MU SVD

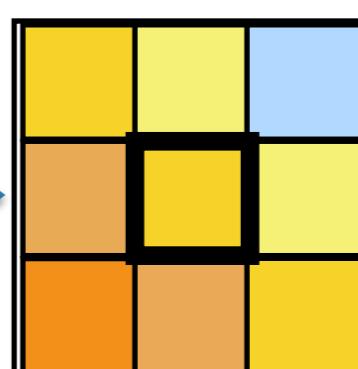
(heterogeneous and non-stationary $\sigma \dot{B}$)



Neighbor



Centered
neighbor



Random
selection

Large scales:

w

Small scales:

$\sigma \dot{B}$

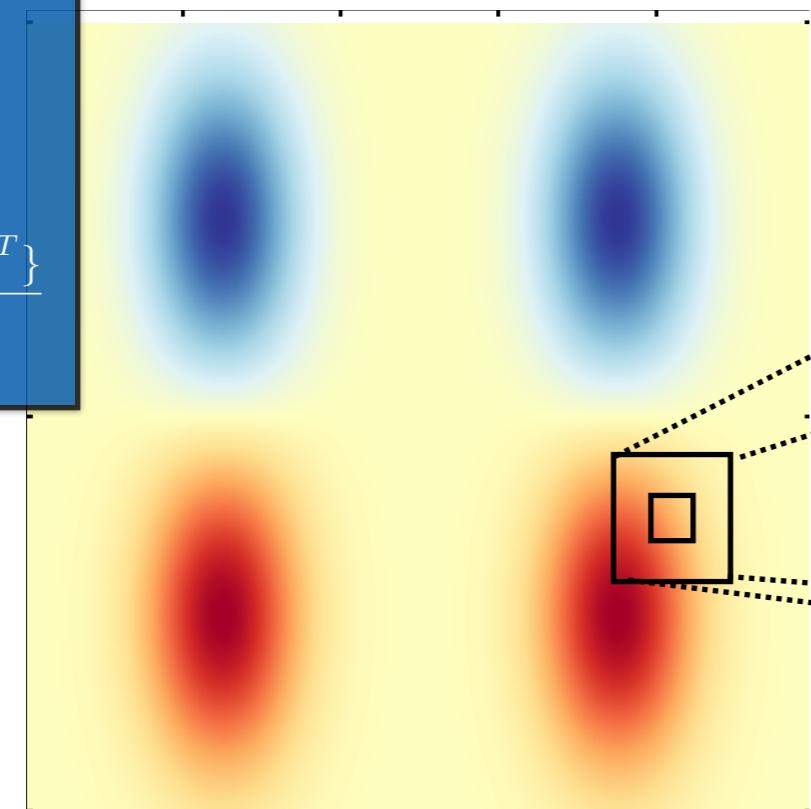
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

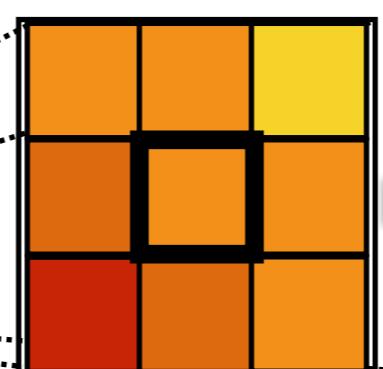
Random switching of points

MU SVD

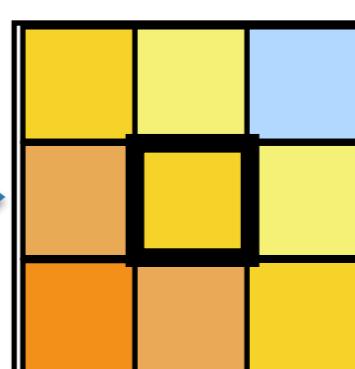
(heterogeneous and non-stationary $\sigma \dot{B}$)



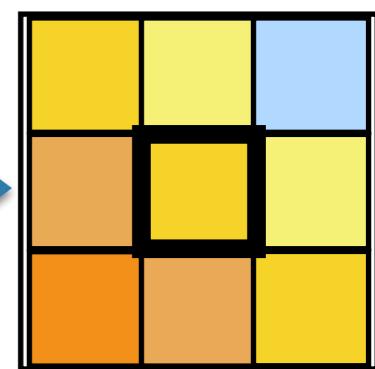
Neighbor



Centered
neighbor



Random
selection



Large scales:

w

Small scales:

$\sigma \dot{B}$

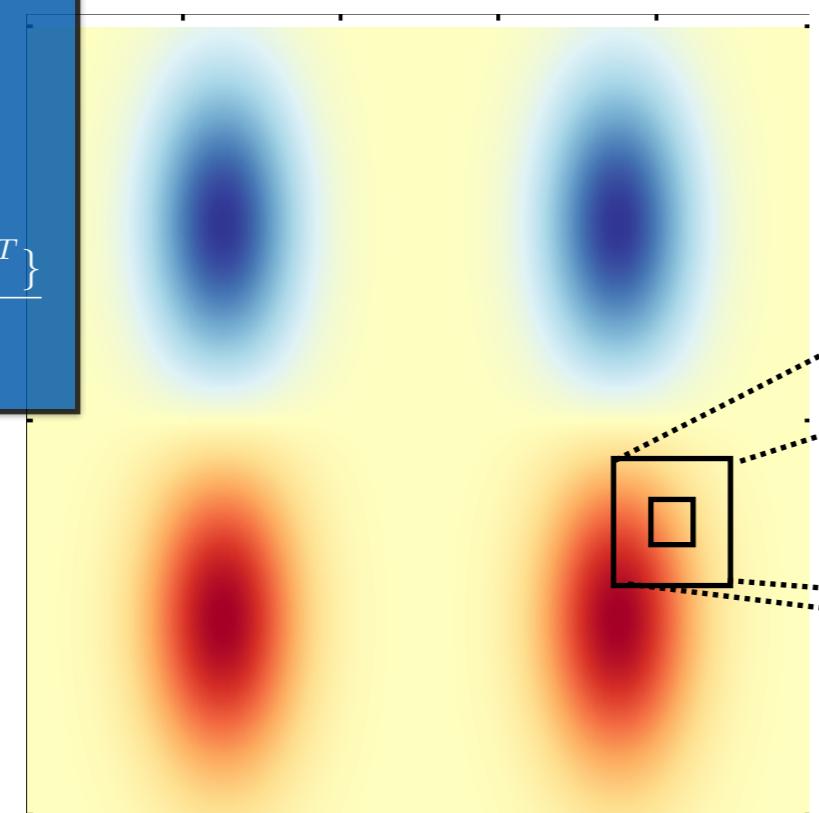
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

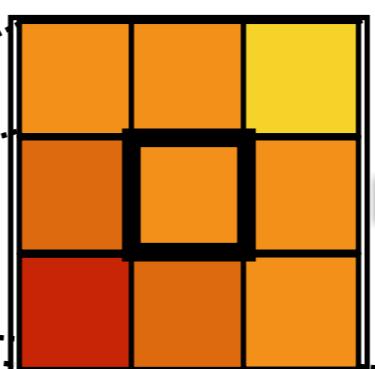
Random switching of points

MU SVD

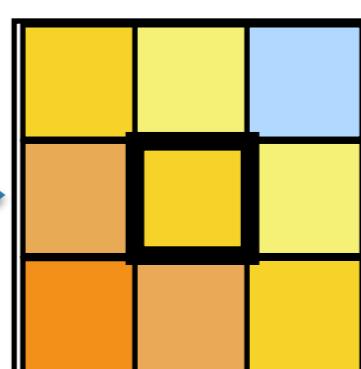
(heterogeneous and non-stationary $\sigma \dot{B}$)



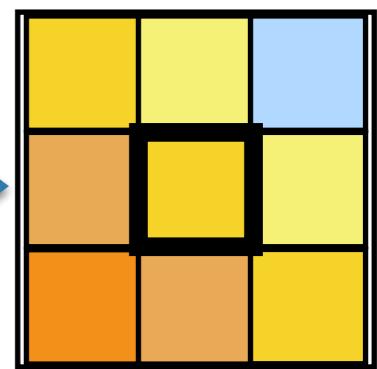
Neighbor



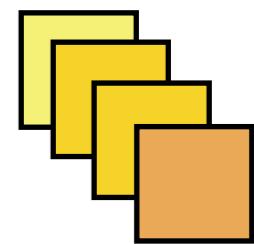
Centered
neighbor



Random
selection



Local ensemble :



Large scales:

w

Small scales:

$\sigma \dot{B}$

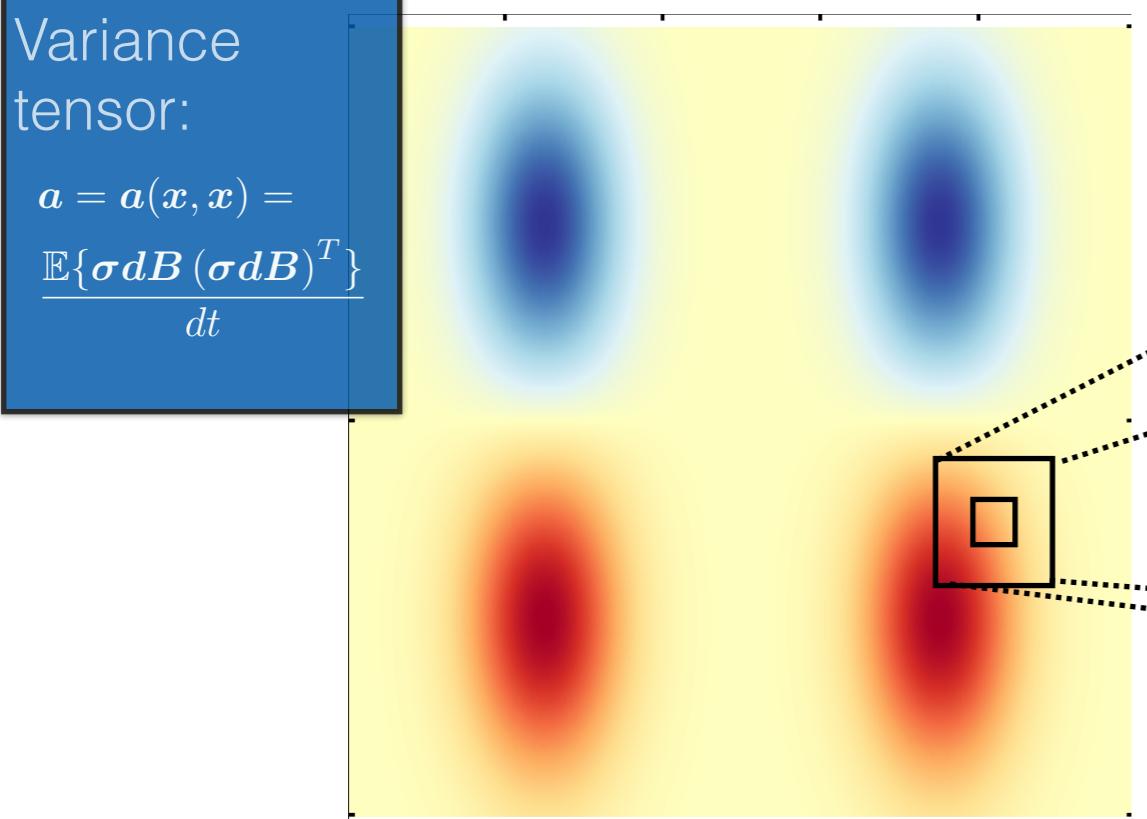
Variance tensor:
 $a = a(x, x) =$

$$\frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

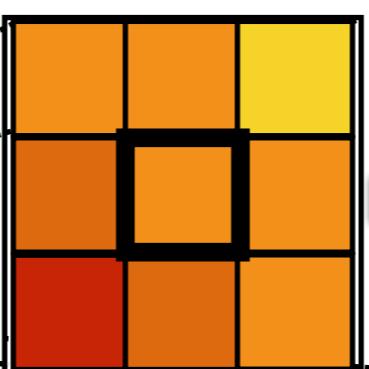
Random switching of points

MU SVD

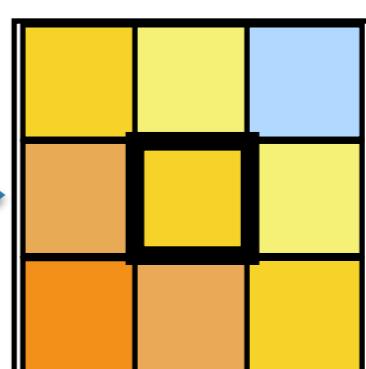
(heterogeneous and non-stationary $\sigma \dot{B}$)



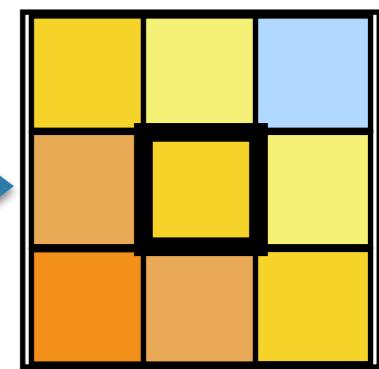
Neighbor



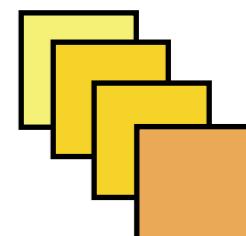
Centered neighbor



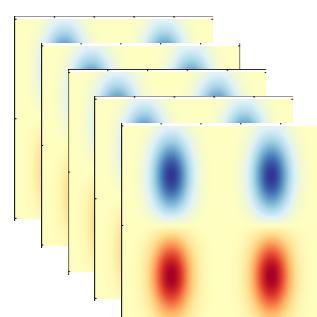
Random selection



Local ensemble :



Global ensemble



Large scales:

w

Small scales:

$\sigma \dot{B}$

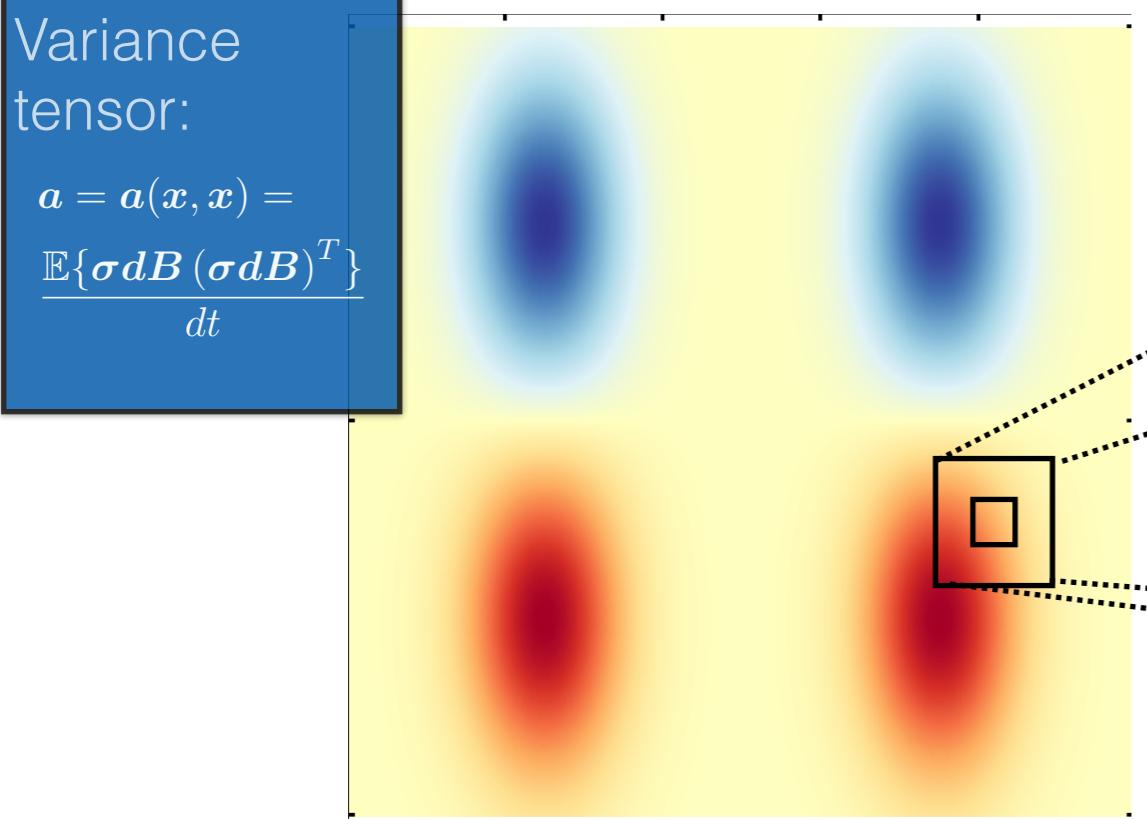
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

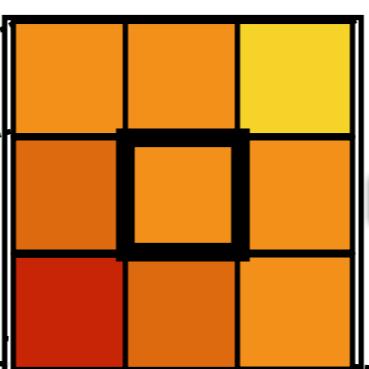
Random switching of points

MU SVD

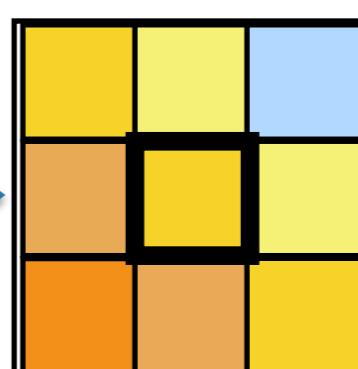
(heterogeneous and non-stationary $\sigma \dot{B}$)



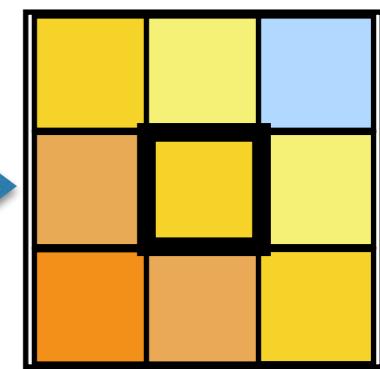
Neighbor



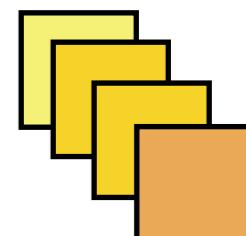
Centered
neighbor



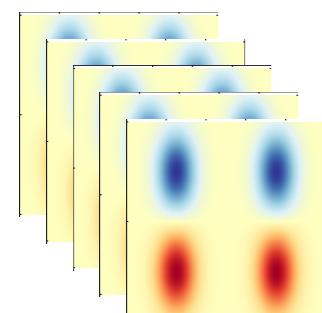
Random
selection



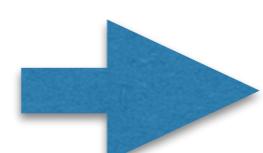
Local ensemble :



Global ensemble



SVD



$\sigma \dot{B}$

Part III

A new energy-budget-based stochastic scheme: WaveHyperv

WaveHyperv

Transport
equation

$$\frac{Dq}{Dt} = \mathcal{L}[q] + \eta$$

WaveHyperv

Transport
equation

$$\frac{Dq}{Dt} = \mathcal{L}[q] + \eta$$

Usual
deterministic
subgrid tensor

e.g. Hyper-viscosity

$$\mathcal{L}[q] = -\nu \Delta^4 q$$

WaveHyperv

Usual
deterministic
subgrid tensor
e.g. Hyper-viscosity

$$\mathcal{L}[q] = -\nu \Delta^4 q$$

Transport equation

$$\frac{Dq}{Dt} = \mathcal{L}[q] + \eta$$



Random forcing
built to meet the energy budget:
(Random energy intake) = $\zeta \times$ Dissipation

Summary of UQ methods

Name	Method
MU Spec	LU with homogeneous and stationary small-scale velocity
MU ADSD	LU with homogeneous, non-stationary and tuning-free small-scale velocity
MU SVD	LU with inhomogeneous and non-stationary small-scale velocity
WaveHyperv	Energy-budget-based stochastic scheme
PIC Spec	Perturbed initial conditions with homogeneous noise
PIC SVD	Perturbed initial conditions with inhomogeneous noise

Part IV

Numerical comparisons

Test case 1:

$t = 17$ days

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

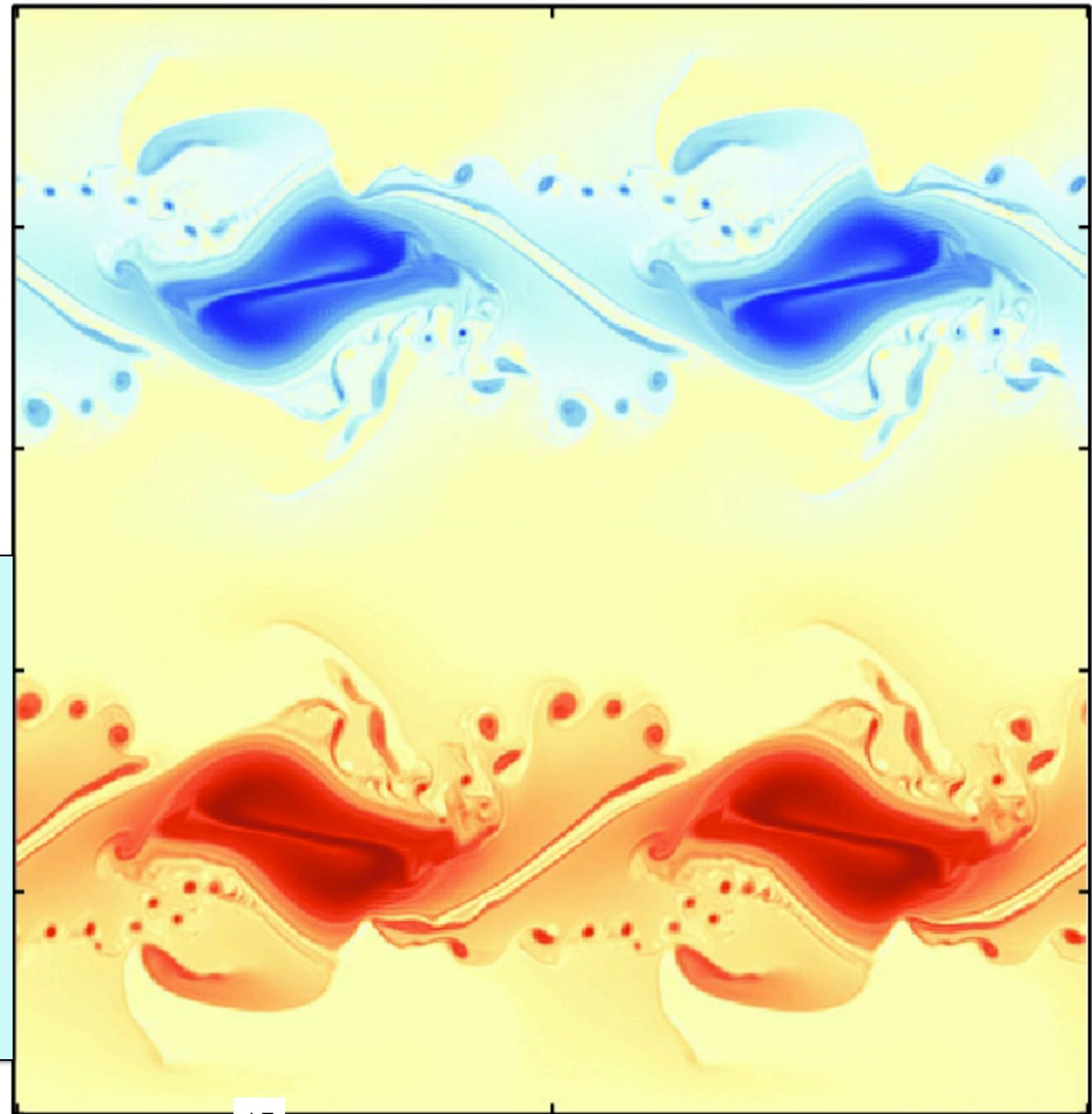
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

512×512



Test case 1:

$t = 17$ days

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

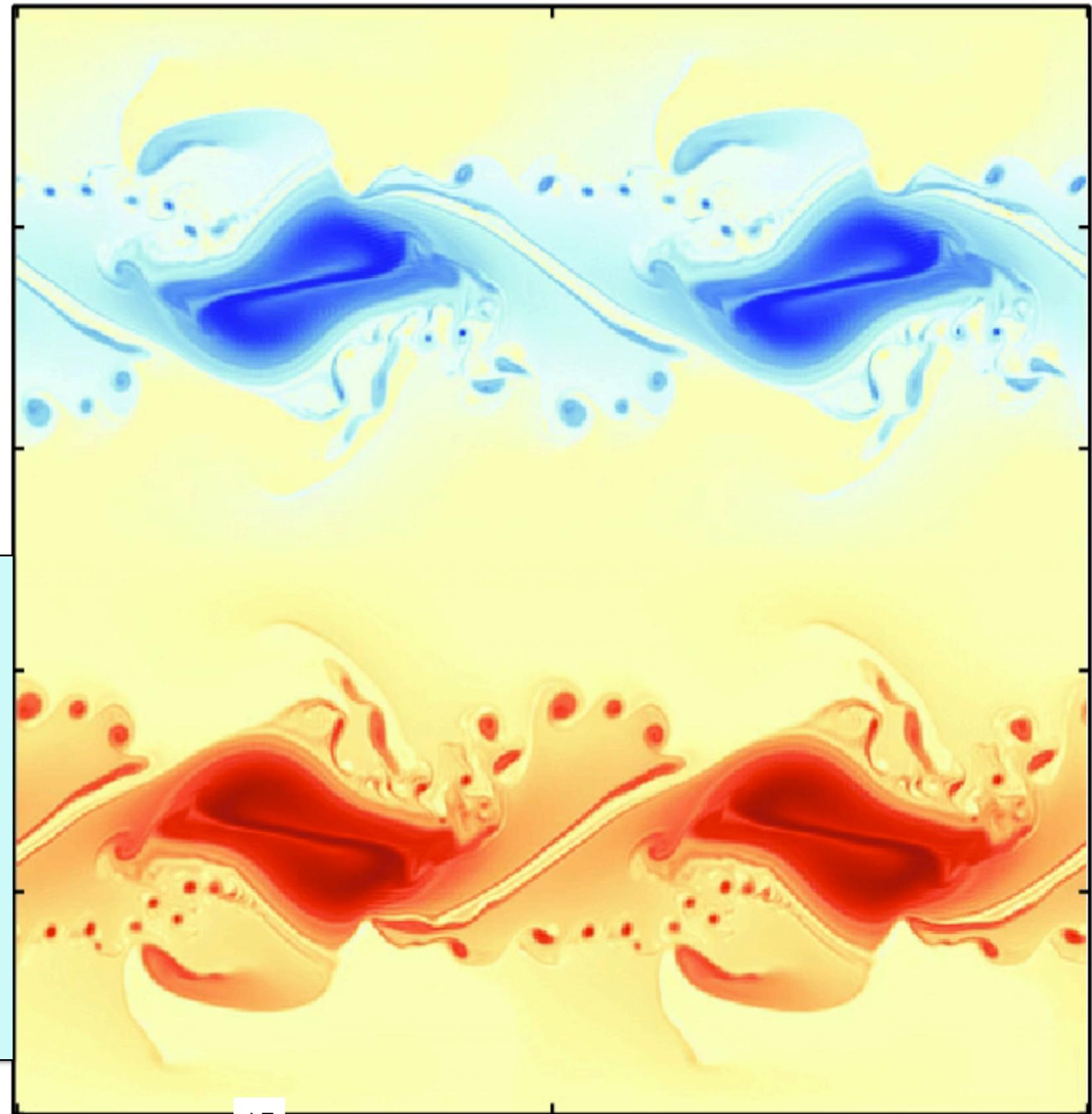
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

512×512



Test case 2:

$t = 10$ days

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

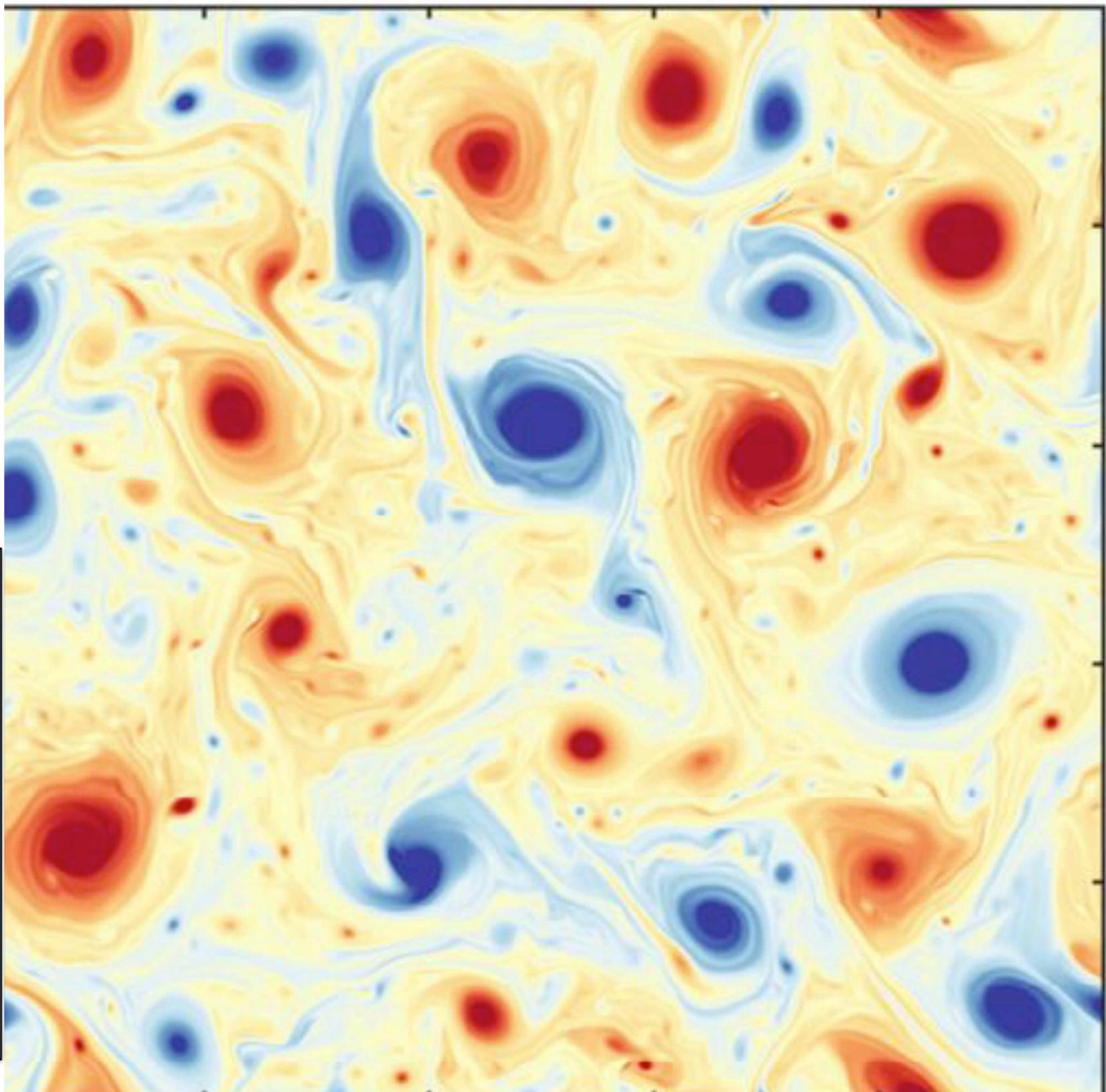
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

512 x 512



Test case 2:

$t = 10$ days

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

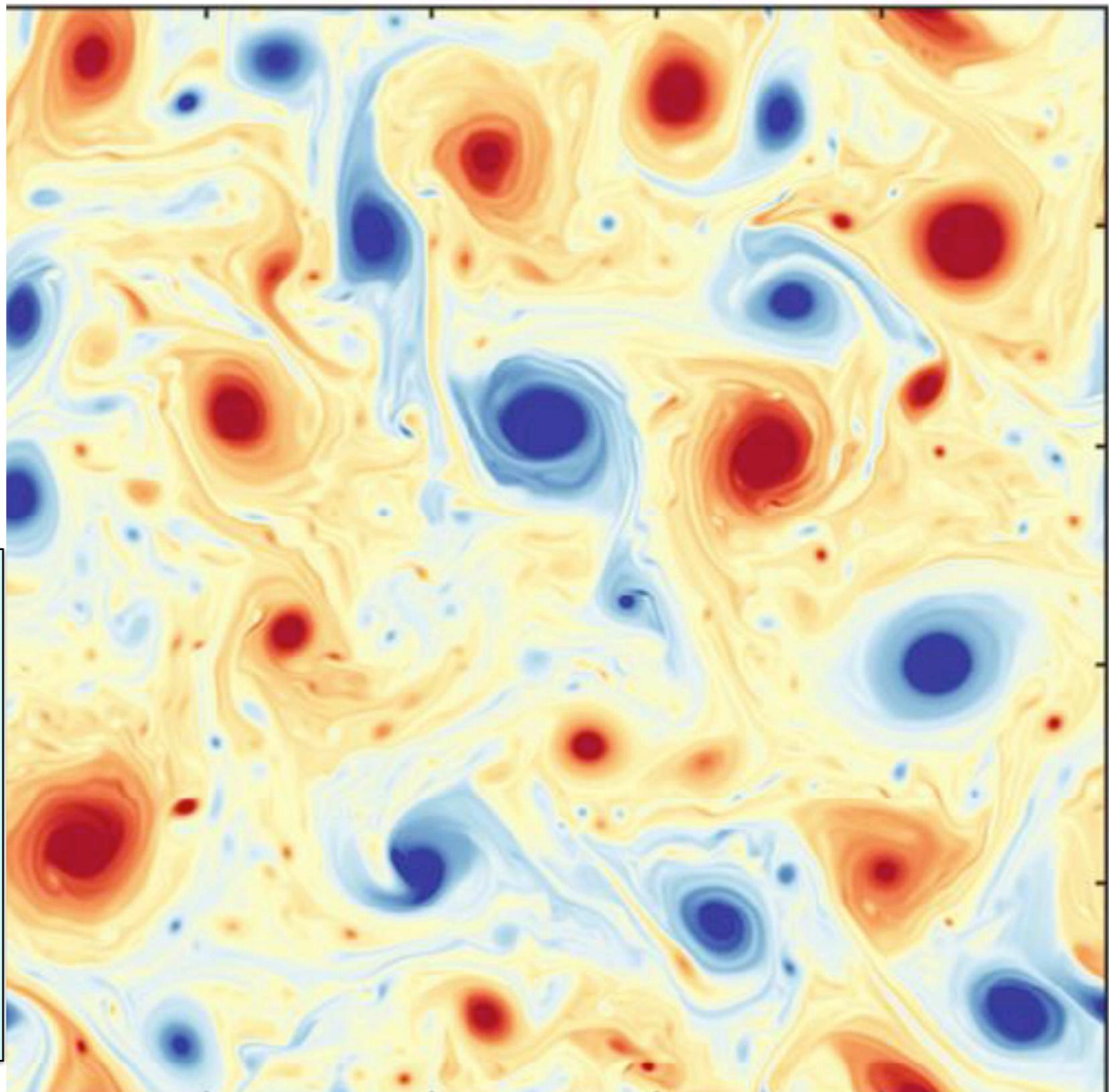
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

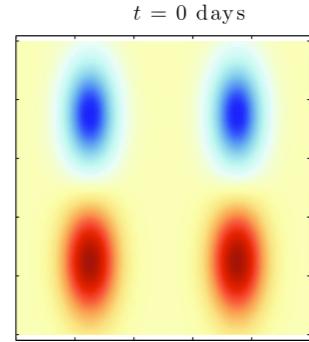
SQG

512 x 512

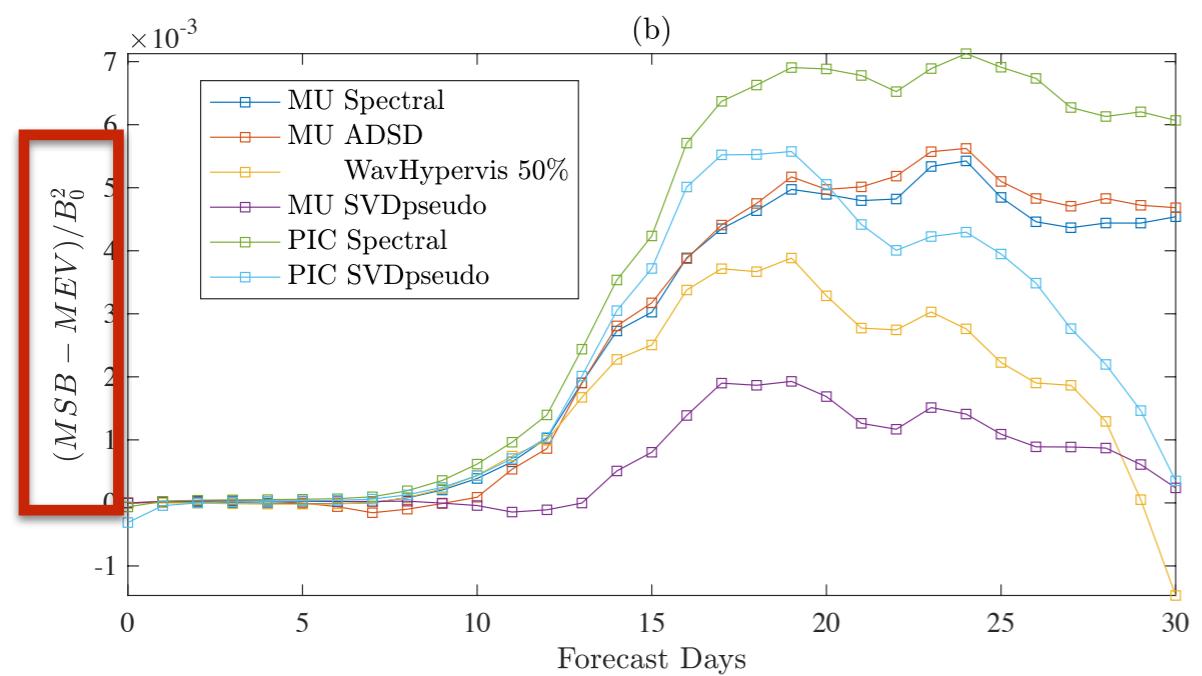
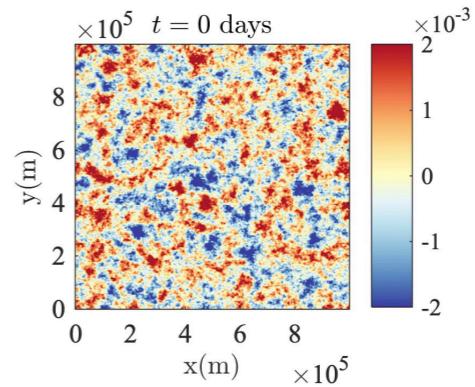


UQ metrics

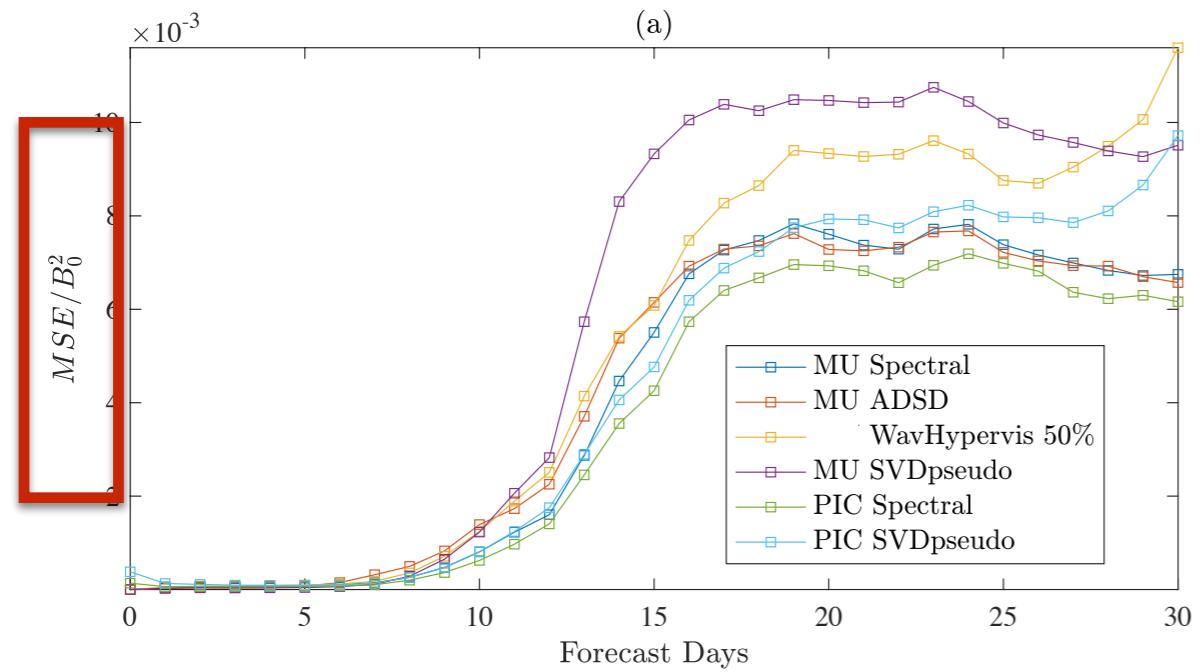
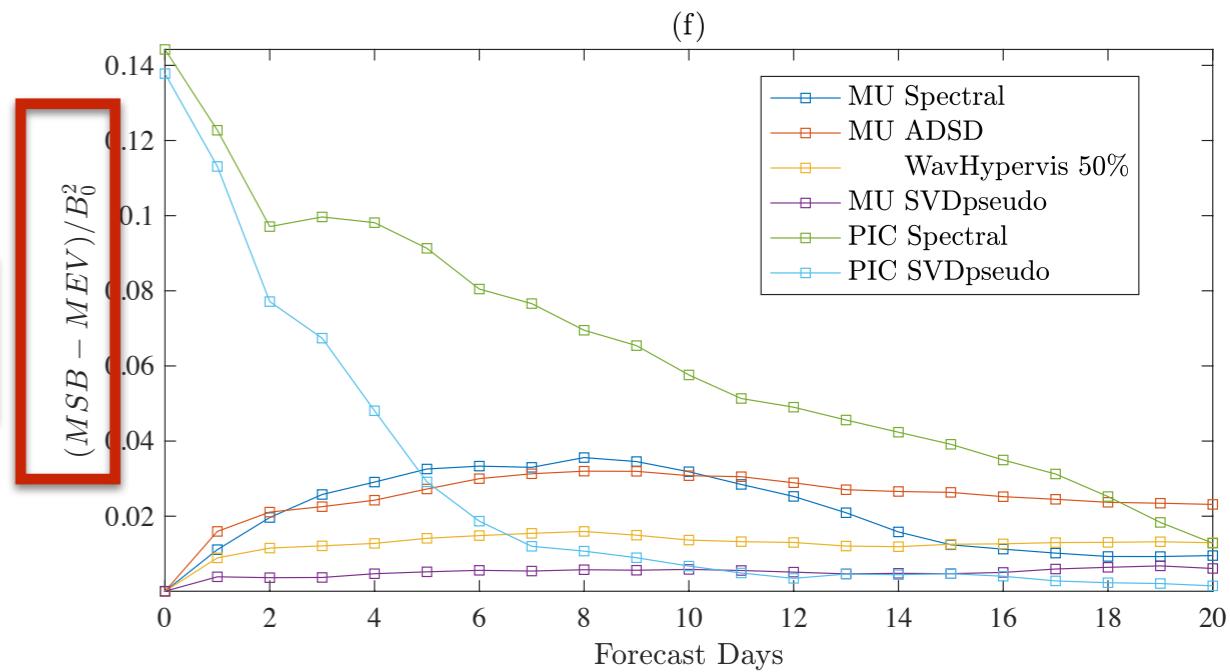
Metric	Meaning
RMSE	<u>Error</u> of ensemble members
Talagrand histogram (TH)	Capacity of the ensemble to <u>explore</u> all reference possible values
Bias^2-spread	Capacity of the ensemble to <u>explore</u> all reference possible values
CRPS	Point-wise distance between the ensemble CFD and the indicator function of the event
Energy Score (ES)	Generalized CRPS for multivariate ensemble



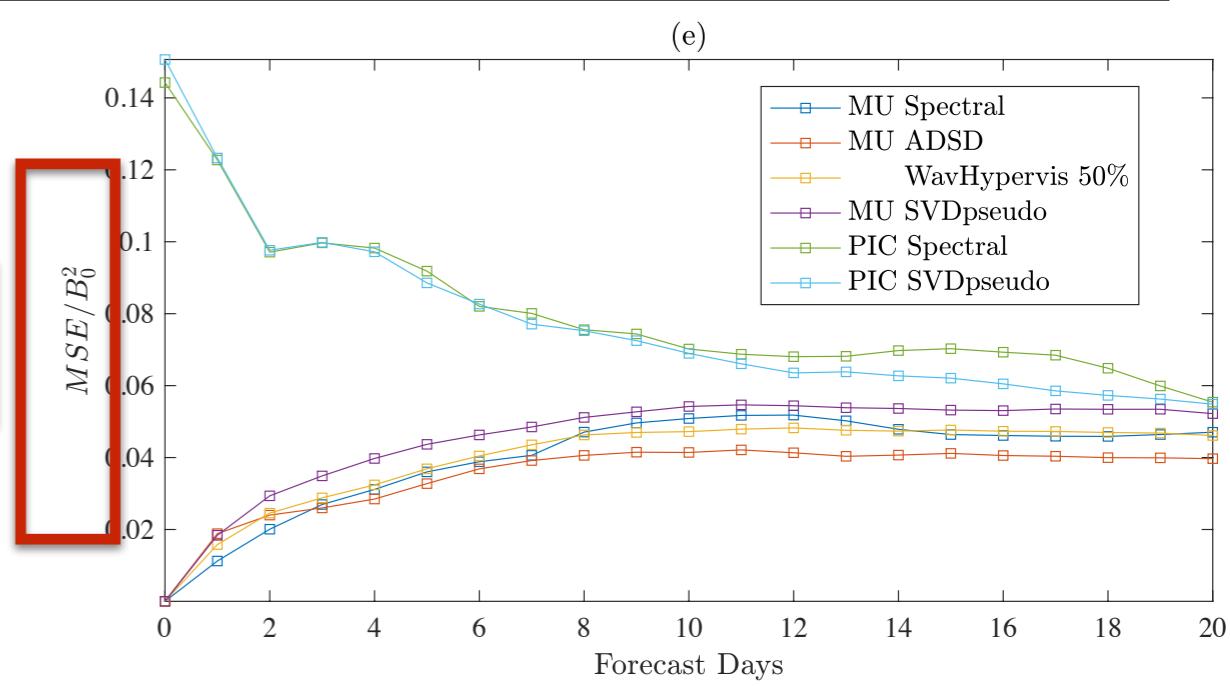
Spreading VS Errors

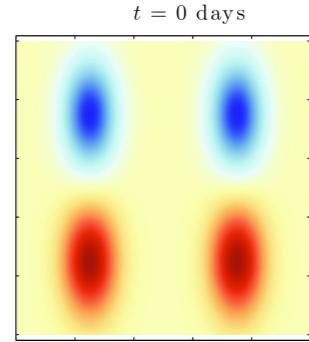


Spread

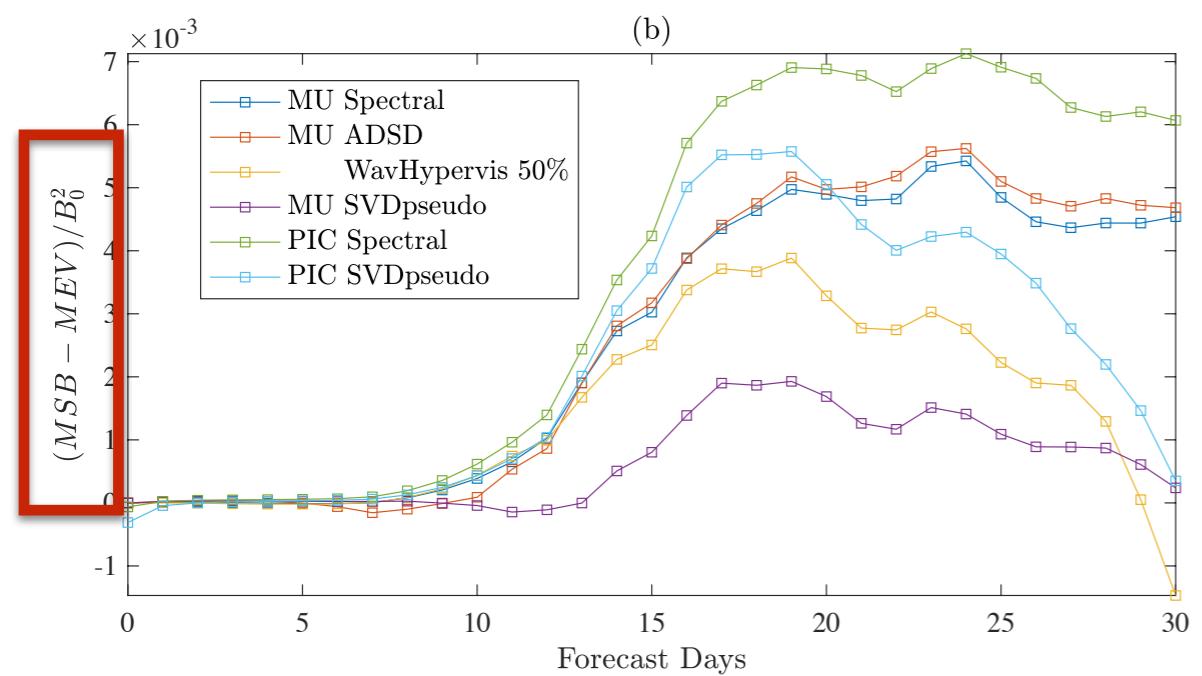
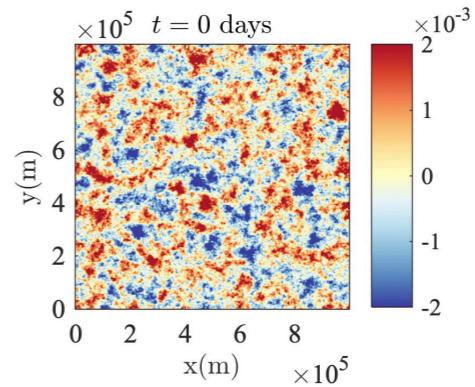


Error

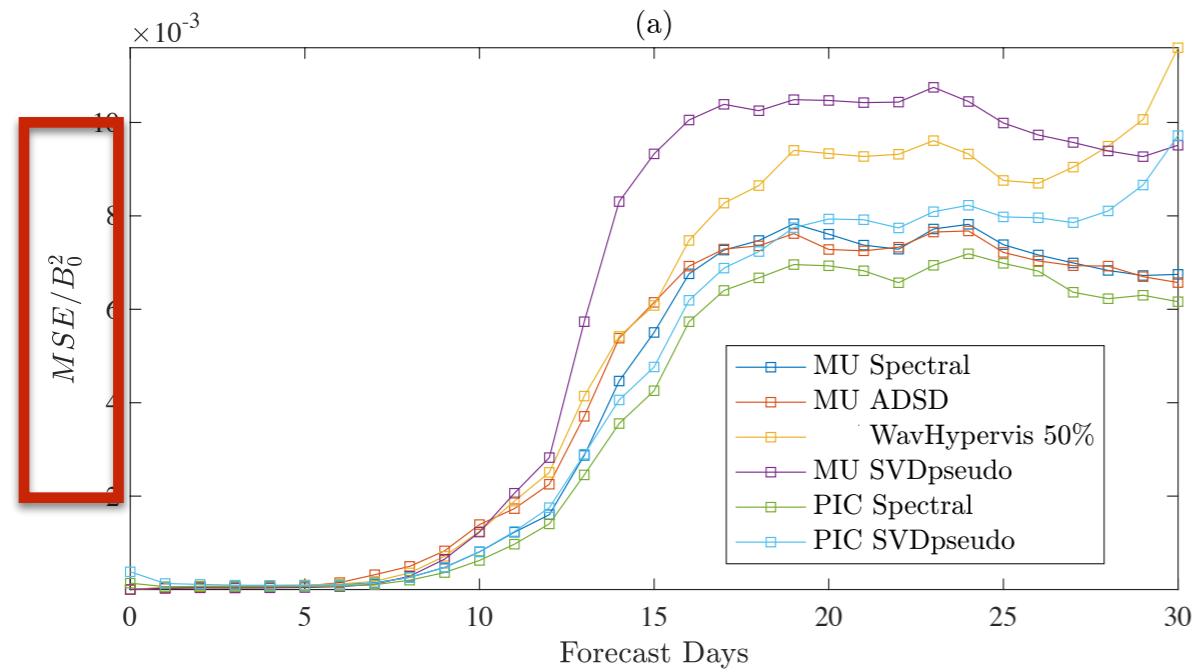
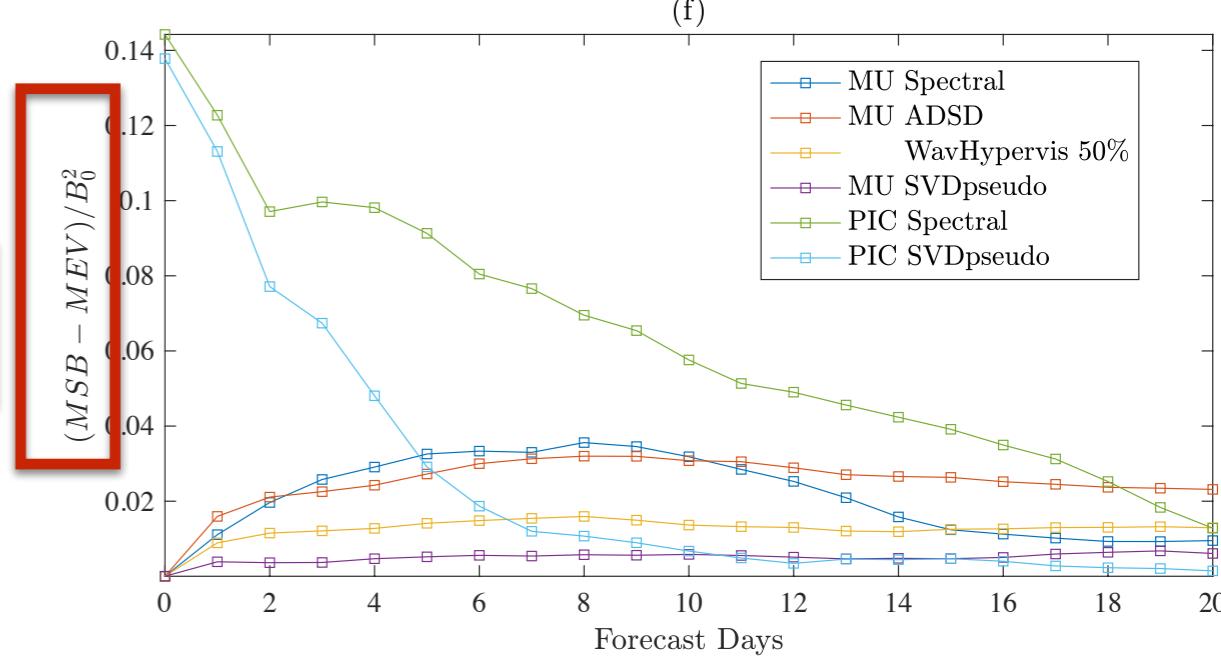




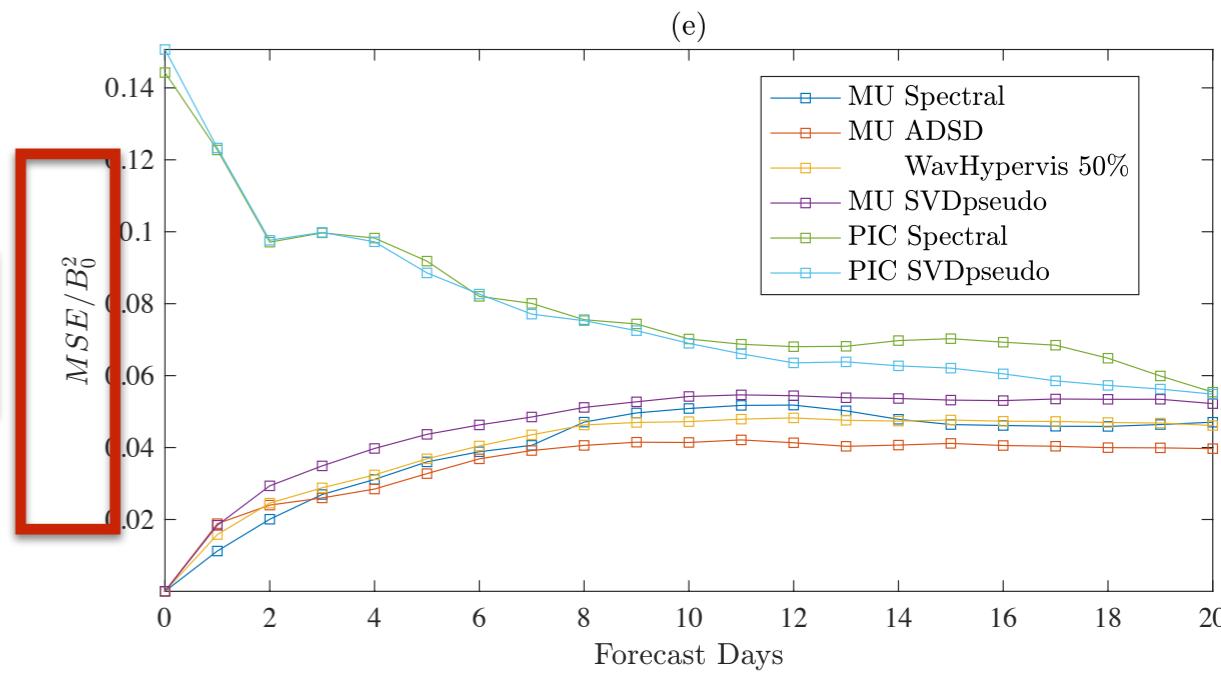
Spreading VS Errors

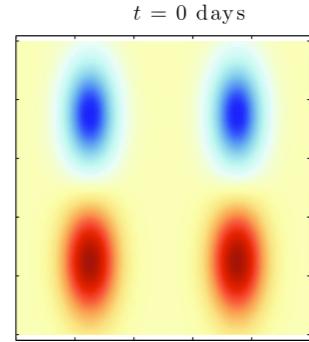


Spread

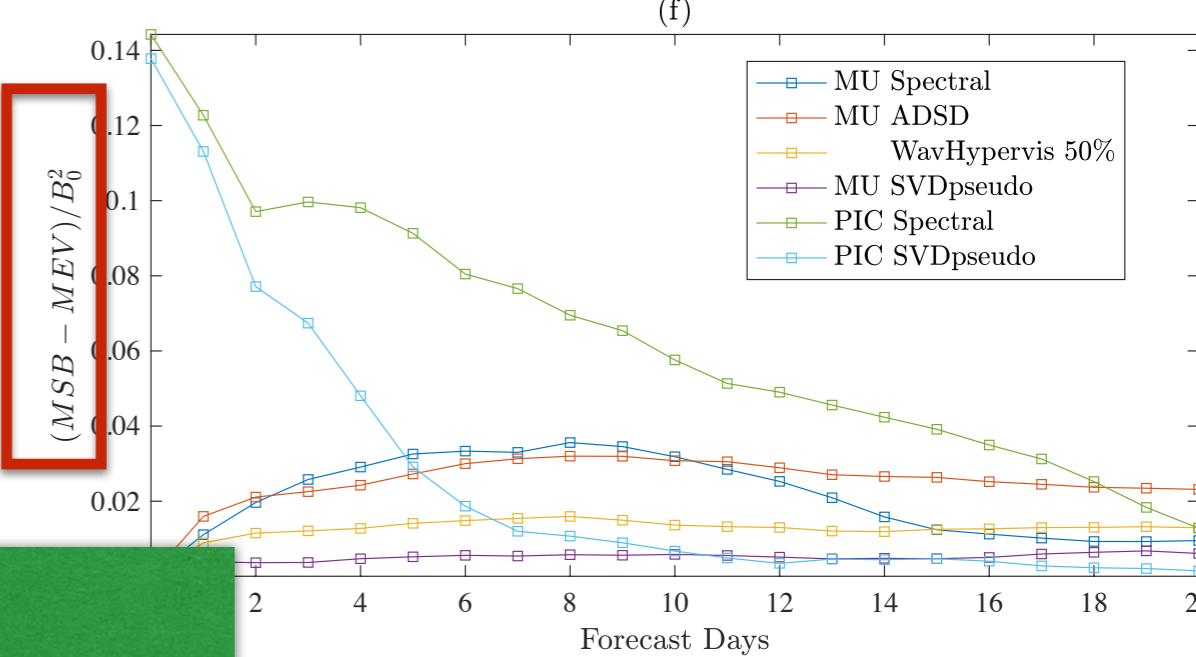
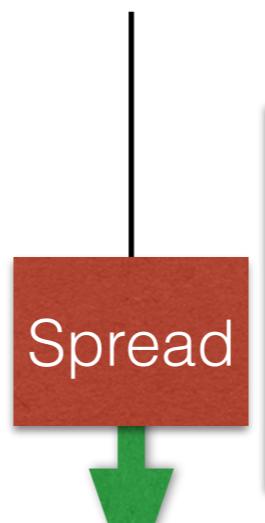
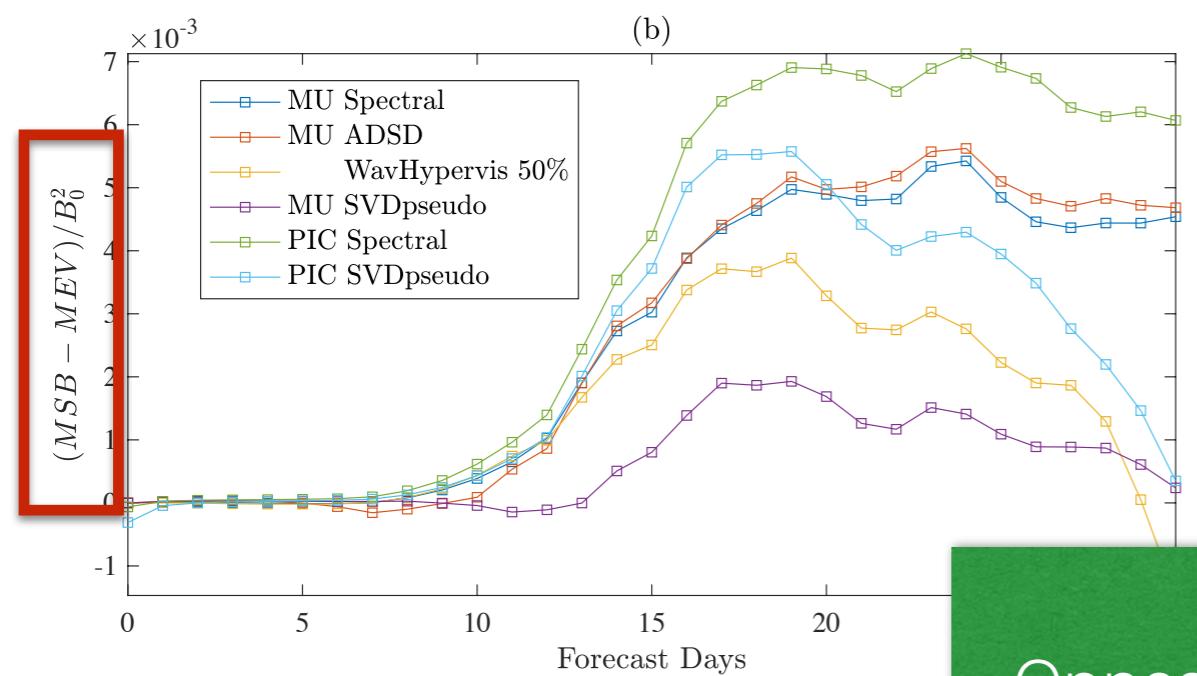
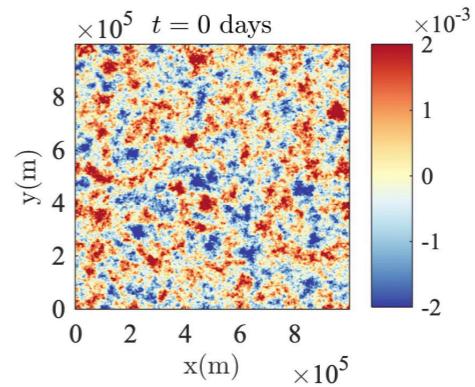


Error

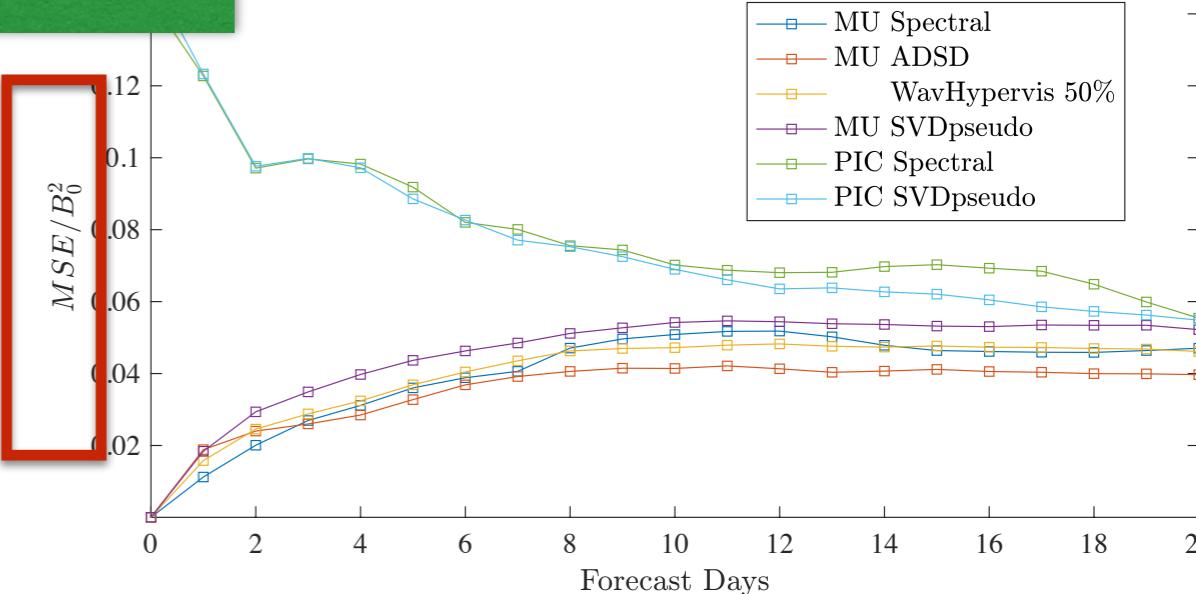
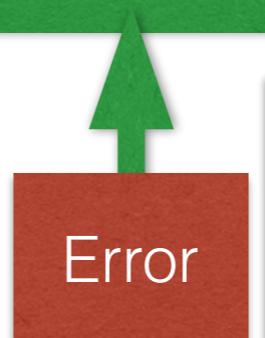
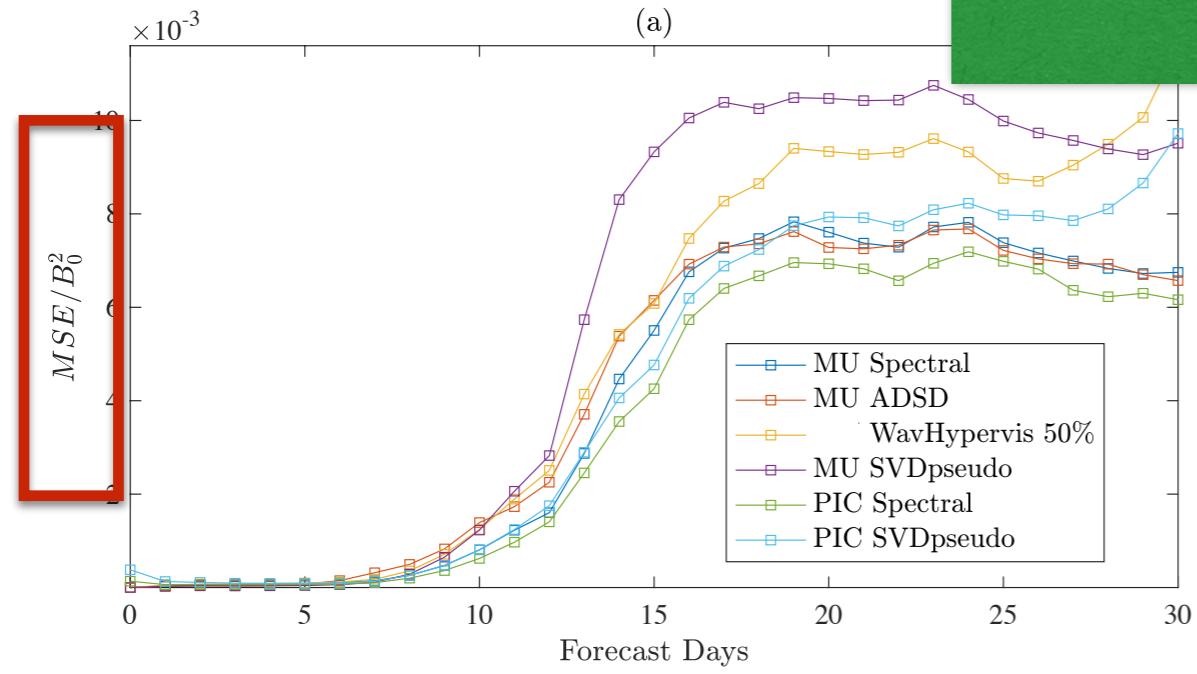




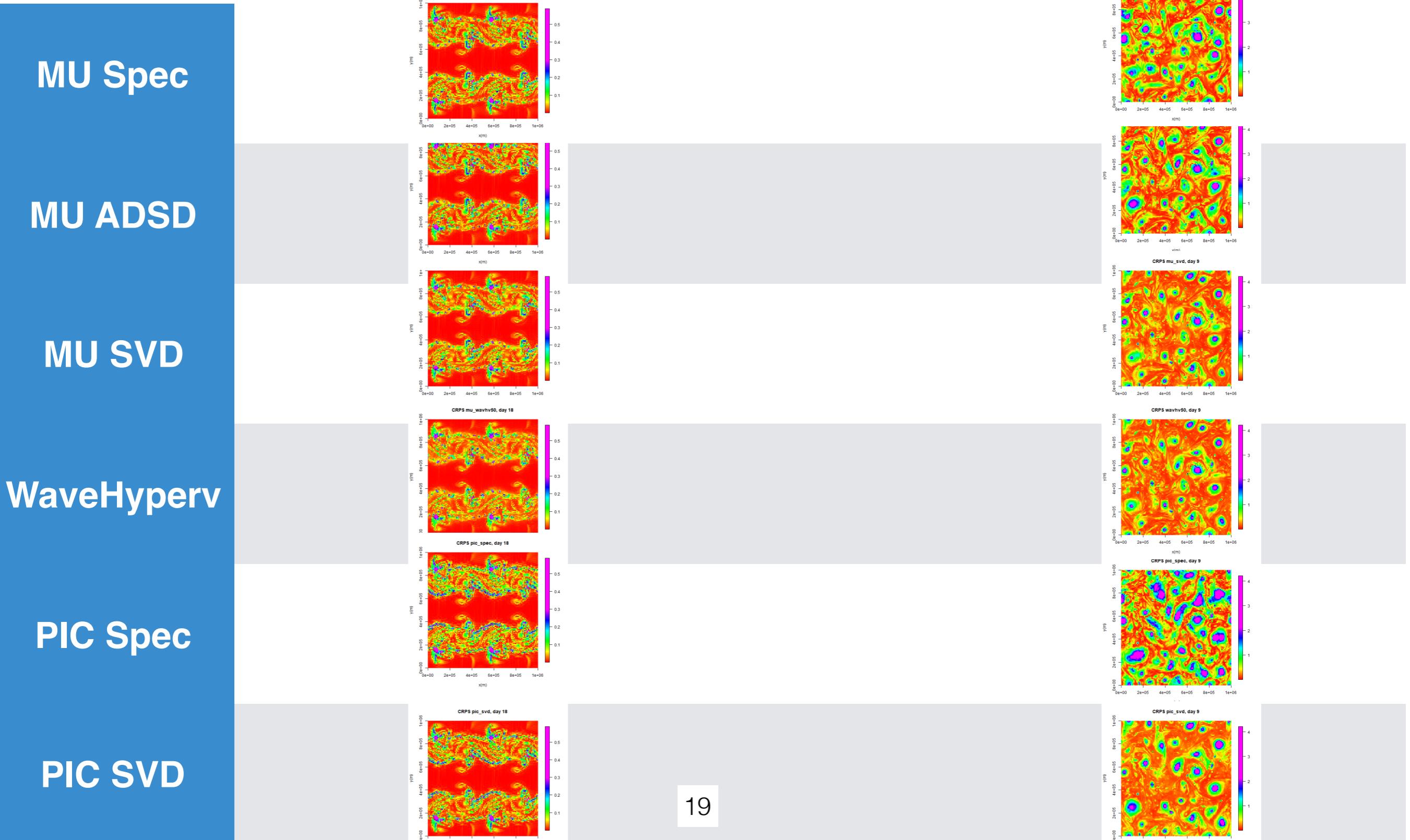
Spreading VS Errors



Opposite conclusions

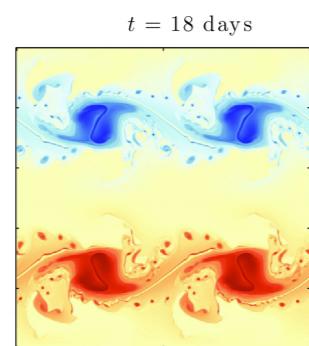


Ensemble point-wise skills : CRPS

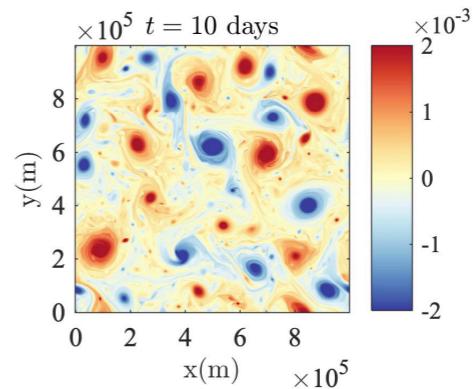


Ensemble point-wise skills : CRPS

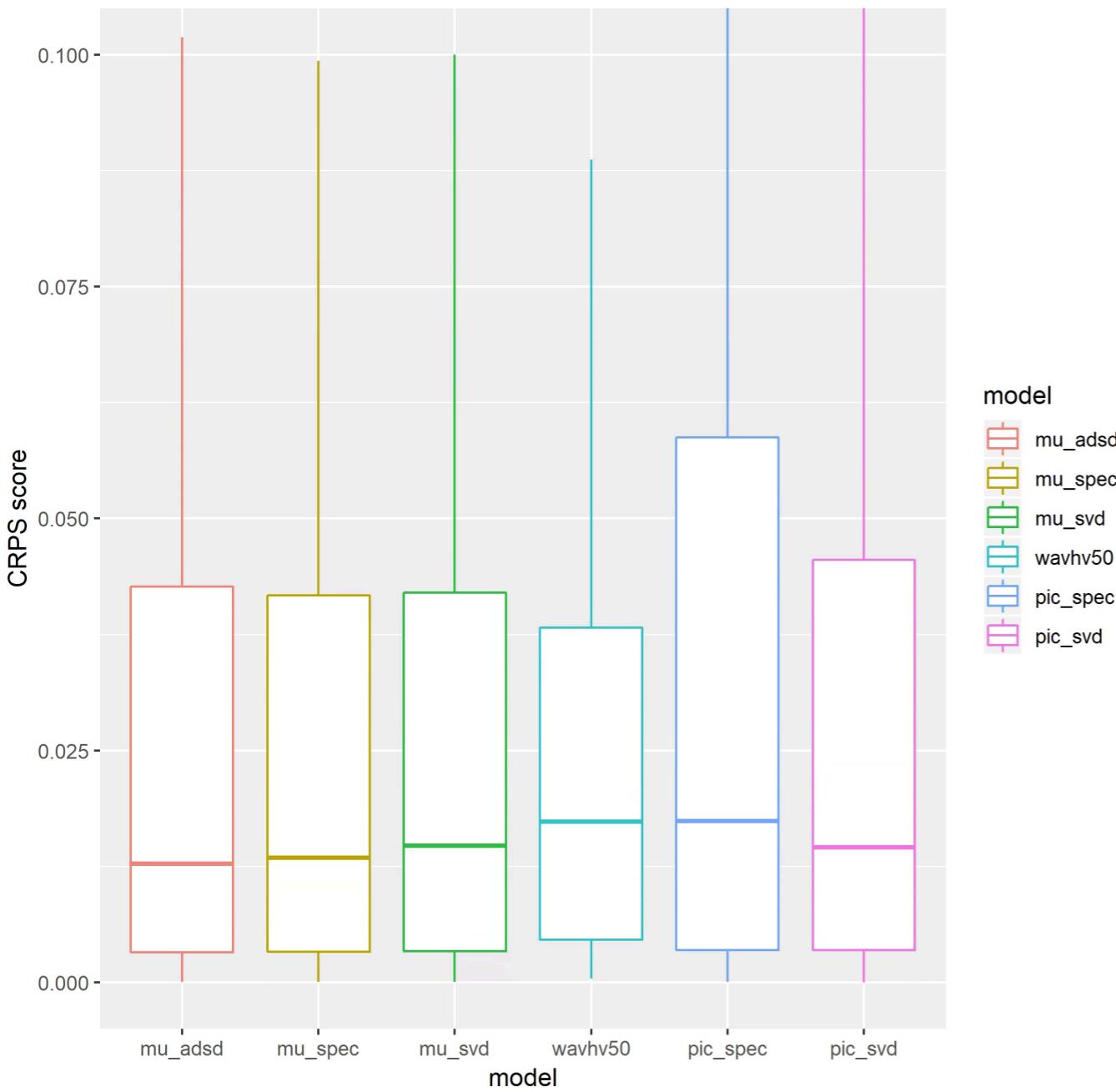




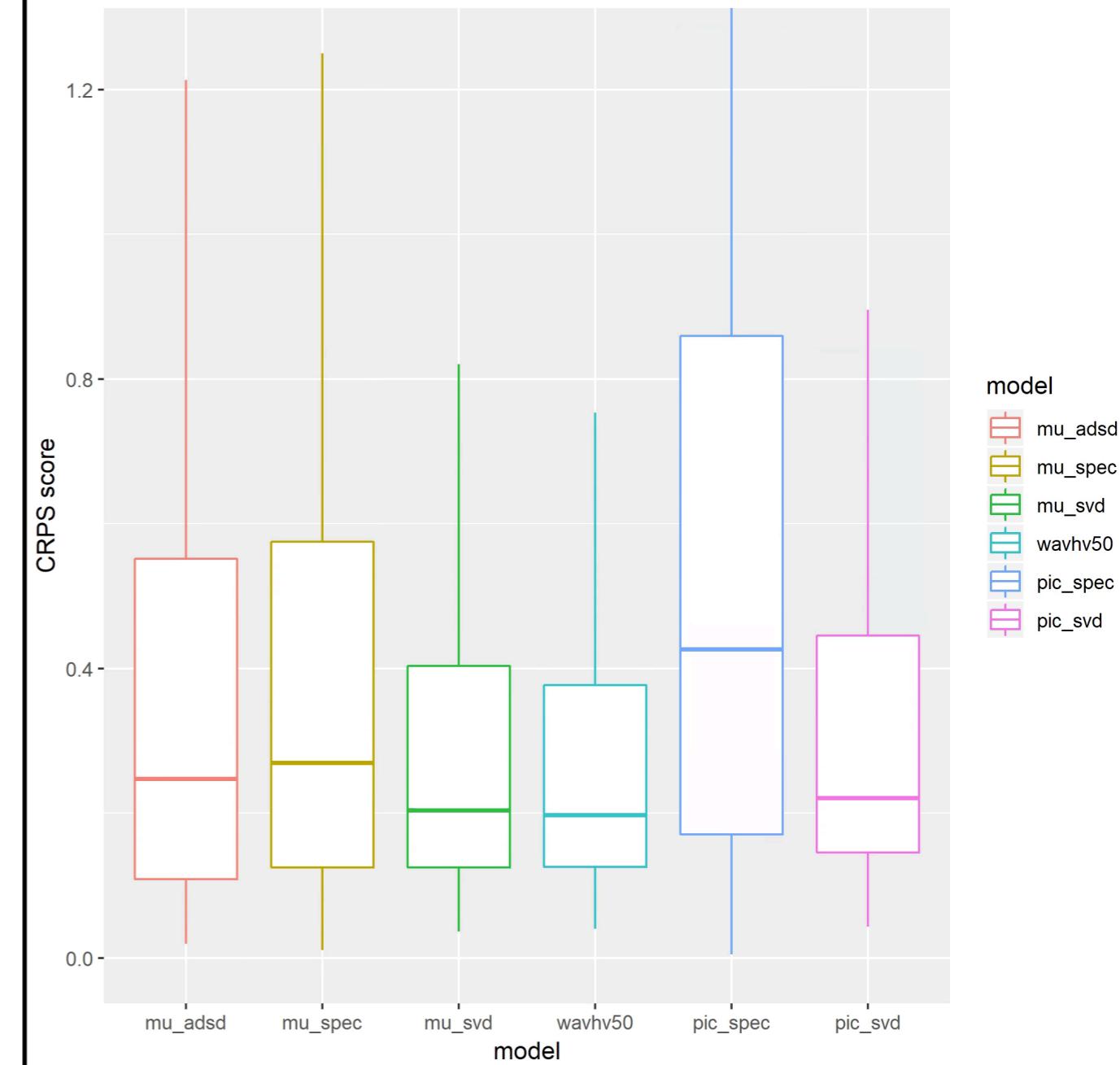
Ensemble point-wise skills : CRPS

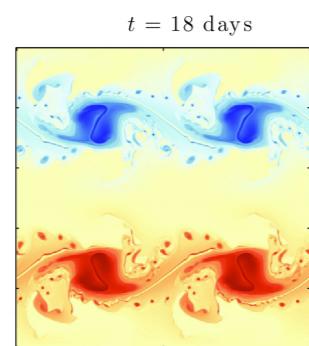


CRPS spatial of each model at day 19

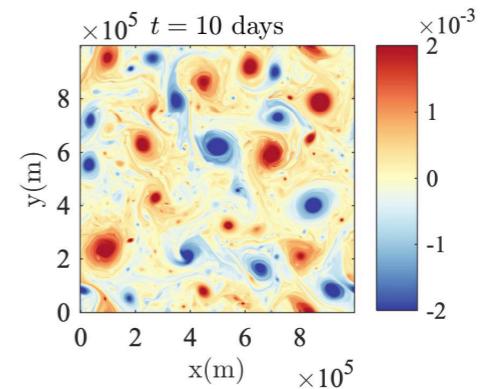


CRPS spatial of each model at day 10

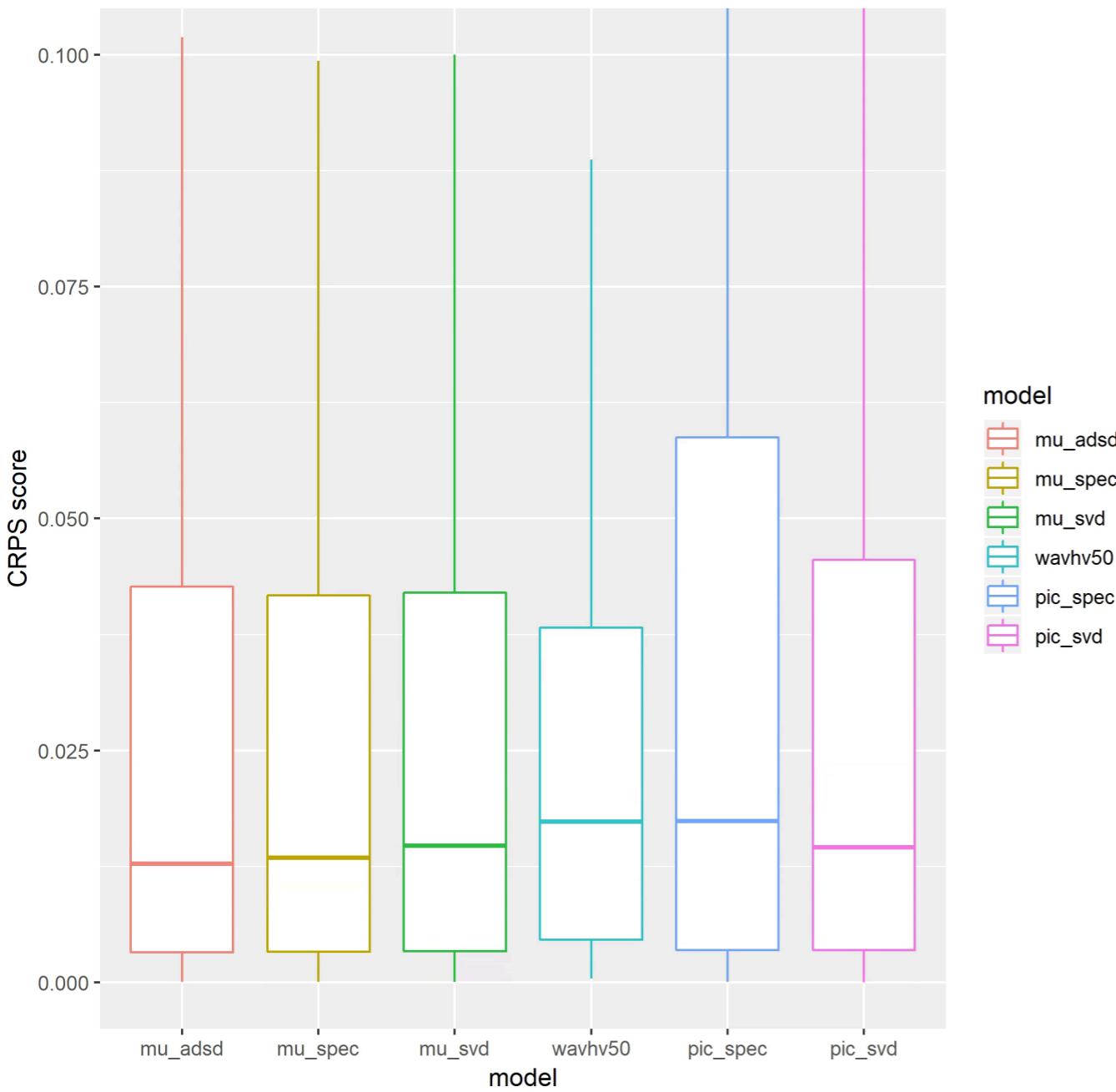




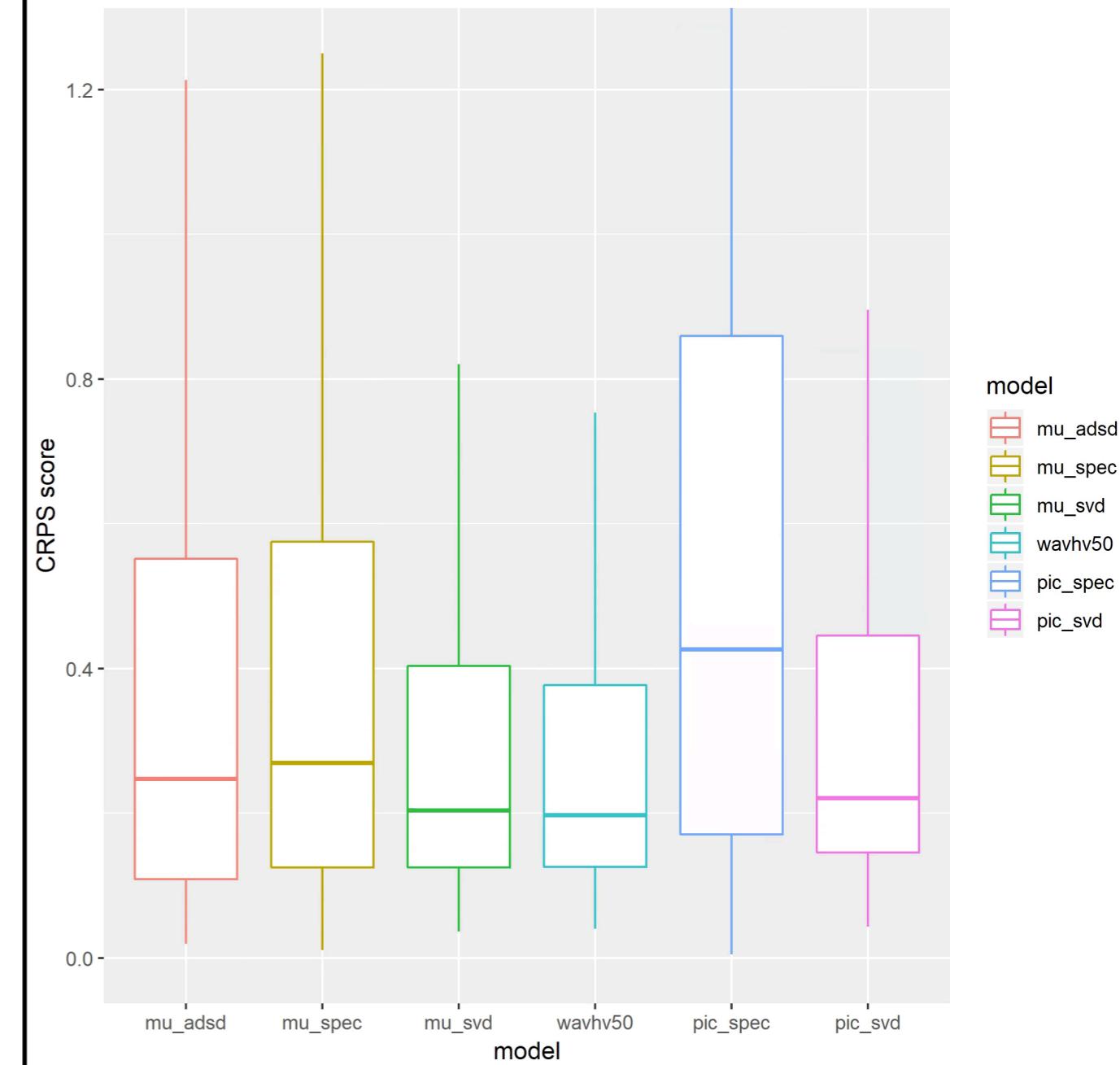
Ensemble point-wise skills : CRPS



CRPS spatial of each model at day 19



CRPS spatial of each model at day 10



Conclusion

Conclusion

	RMSE (errors)	B^2-Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
PIC Spec	-	--	--	--
PIC SVD	--	-	+	+

Conclusion

	RMSE (errors)	B^2-Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
PIC Spec	-	---	---	---
PIC SVD	---	-	+	+

Conclusion

	RMSE (errors)	B^2-Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
PIC Spec	-	--	--	--
PIC SVD	--	-	+	+

Conclusion

	RMSE (errors)	$B^2\text{-Var}$ (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
PIC Spec	-	--	--	--
PIC SVD	--	-	+	+