A consistent framework for stochastic representation of large-scale geophysical flows

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Workshop on Conservation Principles, Data, and Uncertainty in Atmosphere-Ocean Modelling

Introduction

Geophysical flow analysis

- Strong interest on the use of stochastic filters and ensemble methods for data assimilation and forecasting
- Particularly interesting to combine a partially known evolution law with noisy data

⇒ Require stochastic version of the evolution law and/or a modeling of the dynamics errors

Several methodological framework proposed in the literature (Berner et al 2017, Franzke et al. 2015)

- additive/multiplicative forcing (Buiza et al 99), backscatering (Leight 71), (Mason and Thomson 92)
- Low/fast modes decomposition (Majda et al. 99, Franzke et al 05), scale separation (Grooms and Majda).
- Approaches based on stochastic transport (Holm 15, Mémin 14)

Explore expressions of dynamical models under location uncertainties

Location uncertainties

Principle

- Fluid particles displacement can be separated in two components: a smooth differentiable drift $\boldsymbol{w}$ and a random uncertainty function $\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t$
- Flow:

$$\mathbf{X}_t = \mathbf{X}_{t_0} + \int_{t_0}^{t} \boldsymbol{w}(\mathbf{X}_s, s)\mathrm{d}s + \int_{t_0}^{t} \boldsymbol{\sigma}(\mathbf{X}_s, s)\mathrm{d}\mathbf{B}_s,$$

- Displacement:

$$\mathrm{d}\mathbf{X}_t = \boldsymbol{w}(\mathbf{X}_t, t)dt + \boldsymbol{\sigma}(\mathbf{X}_t, t)\mathrm{d}\mathbf{B}_t, \text{ with } \mathbf{X}_{t_0} = \mathbf{x},$$

- Eulerian description of the velocity fields:

$$\mathbf{U}(\mathbf{x}, t) = \boldsymbol{w}(\mathbf{x}, t) + \boldsymbol{\sigma}(\mathbf{x}, t)\dot{\mathbf{B}}_t.$$

- **U** should be solution of Navier Stokes equation derived from Newton 2nd law

Noise term

Uncertainty random field

- **B**: Brownian motion function
- Uncertainty: Diffusion tensor and White noise on Ω

$$(\boldsymbol{\sigma}(\mathbf{x}, t)\mathrm{d}\mathbf{B}_t)^i = \sum_j \int_\Omega \breve{\sigma}^{ij}(\mathbf{x}, \mathbf{y}, t)\mathrm{d}B_t^j(\mathbf{y})\mathrm{d}\mathbf{y}.$$

  Diffusion σ^{ij} Hilbert-Schmidt operator (Covariance of finite norm)

- Covariance of the turbulent component

$$Q_{ij}(\mathbf{x}, \mathbf{y}, t, t') = \sum_k \int_\Omega \breve{\boldsymbol{\sigma}}^{ik}(\mathbf{x}, \mathbf{y}', t)\, \breve{\boldsymbol{\sigma}}^{jk}(\mathbf{y}', \mathbf{y}, t)\mathrm{d}\mathbf{y}'\delta(t - t')\mathrm{d}t$$

$$Q_{ij}(\mathbf{x}, \mathbf{x}, t, t') \triangleq a_{ij}(\mathbf{x})\delta(t - t')\mathrm{d}t$$

Noise term

Example: Kraichnan turbulence model

- Homogeneous diffusion provides homogeneous covariance tensor with spatially constant variance
- Incompressible fluid $\mathrm{d}\boldsymbol{\xi}_t^{\zeta} = \mathcal{P} \star \mathrm{d}\mathbf{B}_t^{\zeta}$
- Spectral correlation defined as

$$\widehat{\mathbf{Q}}(\mathbf{k})_{ij} = |\mathbf{k}|^{-\zeta-d}(\delta_{ij} - \frac{k_i k_j}{|\mathbf{k}|^2})(\widehat{\psi_{\kappa}^{\gamma}})^2 \mathrm{d}t$$

- Quadratic variation process for a passband spectral cutoff ($\mathbb{1}_{[\kappa\gamma]}(\mathbf{k})$)

$$a_{ij} = C_{\zeta,d}\ \zeta^{-1}(L^{\zeta} - \ell_{D}^{\zeta})\delta_{ij}$$

Stochastic Reynolds transport theorem

Volumetric rate of change

Volumetric rate of change of a scalar process $q(\mathbf{x}, t)$ transported by

$$\mathrm{d}\mathbf{X}_t = \boldsymbol{w}(\mathbf{X}_t, t)\mathrm{d}t + \boldsymbol{\sigma}(\mathbf{X}_t, t)\mathrm{d}\mathbf{B}_t, \text{ with } \boldsymbol{\nabla}\cdot\boldsymbol{\sigma} = 0$$

$$\boxed{\mathrm{d}\int_{V(t)} q(\mathbf{x}, t)\mathrm{d}\mathbf{x} = \int_{V(t)} \left(\mathrm{d}_t q + \boldsymbol{\nabla}\cdot\left(q\mathrm{d}\mathbf{X}_t - \frac{1}{2}\boldsymbol{\nabla}\cdot(\boldsymbol{a}q)^{\scriptscriptstyle T}\mathrm{d}t\right)\right)\mathrm{d}\mathbf{x}}$$

Example for the smooth Kraichnan model:

$$\mathrm{d}\int_{\mathcal{V}(t)} q(\mathbf{x}, t)\mathrm{d}\mathbf{x} = \int_{\mathcal{V}(t)} [\mathrm{d}_t q + (\boldsymbol{\nabla}\cdot(q\boldsymbol{w}) - \frac{1}{2}\gamma\Delta q)\mathrm{d}t + \boldsymbol{\nabla}q^{\scriptscriptstyle T}\mathrm{d}\boldsymbol{\xi}_t^{\zeta}]\mathrm{d}\mathbf{x},$$

Stochastic Reynolds transport theorem

Conservation of extensive scalar

Conservation constraint on the transported volume:

$$\mathrm{d}\int_{V(t)} q(\mathbf{x},t)\mathrm{d}\mathbf{x} = 0$$

Evolution law:

$$\mathrm{d}_t q + \boldsymbol{\nabla} q \cdot (\widetilde{\boldsymbol{w}}\mathrm{d}t + \boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t) - \boldsymbol{\nabla}\cdot\left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla} q\right)\mathrm{d}t = q\boldsymbol{\nabla}\cdot\widetilde{\boldsymbol{w}}\mathrm{d}t,$$

$$\mathbb{D}_t q = q\boldsymbol{\nabla}\cdot\widetilde{\boldsymbol{w}}\mathrm{d}t$$

Effective drift: $\widetilde{\boldsymbol{w}} = \boldsymbol{w} - \frac{1}{2}(\boldsymbol{\nabla}\cdot\boldsymbol{a})$

Dissipative operator: $\int_\Omega q\,\boldsymbol{\nabla}\cdot(\boldsymbol{a}\boldsymbol{\nabla} q) = -\sum_{i,j}\int_\Omega \partial_i q\; a^{ij}\partial_j q \leq 0$

Incompressibility: A constant value of the scalar implies

$$\boldsymbol{\nabla}\cdot(\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t) = 0, \quad \boldsymbol{\nabla}\cdot\widetilde{\boldsymbol{w}} = 0$$

Stochastic Reynolds transport theorem

Energy conservation

Energy conservation of transported scalar (incompressible flow)

$$\int_\Omega \mathrm{d}_t(q^2) = 0,$$

Decomposition: $q = \mathbb{E}(q|\widetilde{w}) + \big(q - \mathbb{E}(q|\widetilde{w})\big)$

Energy conservation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbb{E}\|q\|^2_{\mathcal{L}^2(\Omega)} = \frac{\mathrm{d}}{\mathrm{d}t}\|\mathbb{E}(q)\|^2_{\mathcal{L}^2(\Omega)} + \frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega Var(q)\mathrm{d}\mathbf{x} = 0$$

Mean field energy :

$$\partial_t \int_\Omega \big(\mathbb{E}(q)\big)^2 \mathrm{d}\mathbf{x} = -\frac{1}{2}\int_\Omega \boldsymbol{\nabla}\mathbb{E}(q)\boldsymbol{a}\boldsymbol{\nabla}\mathbb{E}(q)\mathrm{d}\mathbf{x} \leq 0.$$

Mean field energy decreases, Variance increases by the same amount

Conservation of momentum

Conservation of momentum

Considering stochastic conservation principle (in a distribution sense)

$$D_t\big(\rho(\boldsymbol{w}(\mathbf{x},t)+\boldsymbol{\sigma}(\mathbf{x},t)\frac{\mathrm{d}\mathbf{B}_t}{\mathrm{d}t})\big)=\boldsymbol{F}(\mathbf{x},t)$$

$\Rightarrow$ **General (incompressible) stochastic Navier-Stokes equations**

$$\mathbb{D}_t(\rho\boldsymbol{w})=-\boldsymbol{\nabla}(p\mathrm{d}t+\mathrm{d}p_t')+\mu\Delta(\boldsymbol{w}\mathrm{d}t+\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t) \qquad \textbf{momentum}$$

$$\mathbb{D}_t\rho=0 \qquad \textbf{mass conservation}$$

$$\boldsymbol{\nabla}\cdot(\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t)=0,\quad \boldsymbol{\nabla}\cdot\boldsymbol{w}-\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\mathbf{a})=0 \qquad \textbf{incompressibility}$$

with

$$\mathbb{D}_t q=\mathrm{d}_t q+\boldsymbol{\nabla}q\boldsymbol{\cdot}\big(\widetilde{\boldsymbol{w}}\mathrm{d}t+\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t\big)-\frac{1}{2}\boldsymbol{\nabla}\boldsymbol{\cdot}(\boldsymbol{a}\boldsymbol{\nabla}q)\mathrm{d}t$$

Navier Stokes equations - LES\LU

LES under location uncertainty

- Large scale drift of finite variation $\Rightarrow$ separation of Brownian terms and finite variation terms

Momentum equations

$$\partial_t \boldsymbol{w} + (\widetilde{\boldsymbol{w}} \cdot \boldsymbol{\nabla})\boldsymbol{w} - \frac{1}{2}\boldsymbol{\nabla} \cdot (\tau) = -\frac{1}{\rho}\boldsymbol{\nabla} p + \nu \Delta \boldsymbol{w},$$

Effective drift

$$\widetilde{\boldsymbol{w}} = \left(\boldsymbol{w} - \frac{1}{2}(\boldsymbol{\nabla} \cdot \boldsymbol{a})^{T}\right)$$

Subgrid tensor

$$\tau = ((a\boldsymbol{\nabla})\boldsymbol{w})$$

Pressure random contribution

$$\frac{1}{\rho}\boldsymbol{\nabla} \mathrm{d}\hat{p}_t = -(\boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t \cdot \boldsymbol{\nabla})\boldsymbol{w} + \nu \Delta(\boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t),$$

Mass conservation

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t) = 0, \ \boldsymbol{\nabla} \cdot \boldsymbol{w} - \frac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \cdot \boldsymbol{a} = 0$$

Navier Stokes equations - LES\LU

Incompressible case

Subgrid model:

- Constant eddy viscosity model for Kraichnan uncertainty model
- Smagorinsky model for $\mathbf{a} = c\|\boldsymbol{S}\|\mathbb{I}$ with $\|\boldsymbol{S}\|$ smooth enough
- Noise on isopycnal surfaces $\Longrightarrow$ Gent-McWilliams
- New LES schemes with $\boldsymbol{a}$ defined from local variance
- Stochastic models from POD noise on high resolution simulations, resolved fluctuations, predefined covariances, targeted dissipation (hyperviscosity), etc.

- **Simulations**: Wake flows, channel flows, Green-Taylor flow, geophysical flow approximations

Geophysical flow modelling

Geophysical flow modelling

- Same derivation and assumption as in the deterministic case
- Stratification, traditional approximation, etc.
- Introduction of the transport stochastic operator (material derivative)
- Noise specification for the numerical simulations

Geophysical flow modeling

Simple Boussinesq equations (LU)

Momentum equations

$$\mathbb{D}_t \boldsymbol{w} + f\,\boldsymbol{k} \times (\boldsymbol{u} + (\boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t)_H) = b\;\boldsymbol{k} - \frac{1}{\rho_b}\boldsymbol{\nabla} p' + \mathcal{F}(\boldsymbol{w}),$$

Effective drift

$$\widetilde{\boldsymbol{w}} = \begin{pmatrix} \widetilde{\boldsymbol{u}} \\ \widetilde{w} \end{pmatrix} = \boldsymbol{w} - \frac{1}{2}(\boldsymbol{\nabla} \cdot \boldsymbol{a})^{T},$$

Buoyancy equation

$$\mathbb{D}_t b + N^2 \left(\widetilde{w} \mathrm{d}t + (\boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t)_z\right) = \frac{1}{2}\boldsymbol{\nabla} \cdot \left(\boldsymbol{a}_{\bullet z} N^2\right) \mathrm{d}t,$$

$$b = -\frac{g}{\rho_b}\rho', N^2(z) = -\frac{g}{\rho_b}\partial_z \rho_0(z)$$

Incompressibility

$$\boldsymbol{\nabla} \cdot \boldsymbol{w} - \frac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \cdot \boldsymbol{a} = 0; \quad \boldsymbol{\nabla}\cdot\left(\boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t\right) = 0$$

Geophysical flow modelling

Stochastic approximated model

- Zoology of approximated models obtained from power series expansion in small number (Rossby) and scaling
- PG, QG, SQG, ...
- Same strategy and Proper scaling of the noise term
- $\Rightarrow$ Stochastic PG, QG, SQG, ... see talks of Long Li and Valentin Resseguier
  - Global energy conservation, Modified enstrophy conservation for general noises
  - Structure preservation

Geophysical flow modelling

Usefulness of physically consistant Stochastic modeling

- Accuracy of large-scale models and error representation
- Fast exploration of attractor's regions
- New physical modeling capabilities

Geophysical flow modelling

Usefulness of physically consistant Stochastic modeling

- **Accuracy of large-scale models and error representation**
- **Fast exploration of attractor's regions**
- New physical modeling capabilities

Geophysical flow modelling

Stochastic Lorenz model (LU) - Chapron et al QJRMS 2017

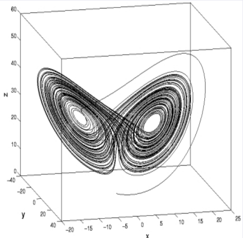
- Rayleigh-Bénard convection (incompressible fluid with $T_b > T_u$)
- Galerkin projection on Fourier modes of Navier-Stokes with Boussinesq approximation

$$\frac{\mathrm{d}X}{\mathrm{d}t} = P_r(Y - X) - \frac{4}{2\Upsilon}X$$

$$\mathrm{d}Y = [X(\rho - Z) - Y - \frac{4}{2\Upsilon}Y]\mathrm{d}t + \frac{\rho - Z}{\Upsilon^{1/2}}\mathrm{d}\mathbf{B}_t$$

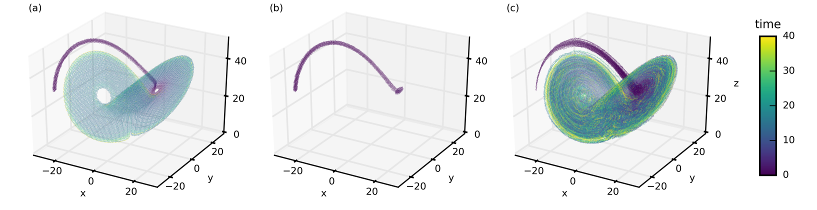
$$\mathrm{d}Z = [XY - bZ - \frac{8}{2\Upsilon}Z]\mathrm{d}t + \frac{Y}{\Upsilon^{1/2}}\mathrm{d}\mathbf{B}_t$$

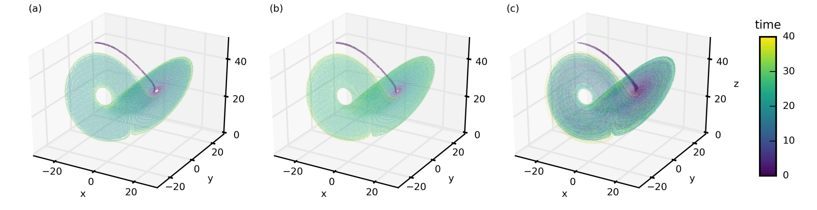
Usual model $\sim$ DNS
Diffusive model $\sim$ LES
Model under uncertainty



Geophysical flow modelling

Attractor visits exemple (strong noise and low noise)

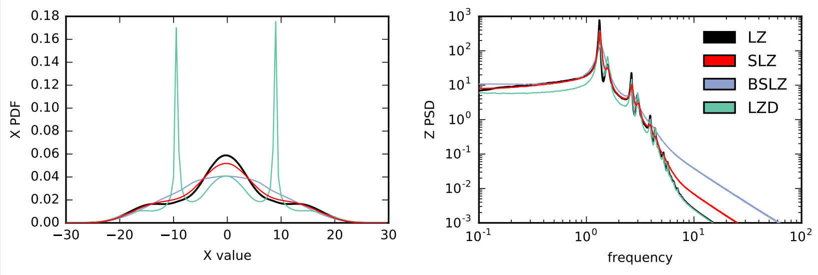




Points of the Lorenz attractor visited (ensemble of 100 particles)
LZ (a), LZD (b), SLZ (c); colors: time of the visit; $\Upsilon = 10$; $\Upsilon = 100$

Geophysical flow modelling

Empirical PDF of X and spectrum of Z strong noise $\Upsilon = 10$



10.000 realizations $t \in [0, 100]$

BSLZ: adhoc stochastic Lorenz with multiplicative noise (Chekroun et al)

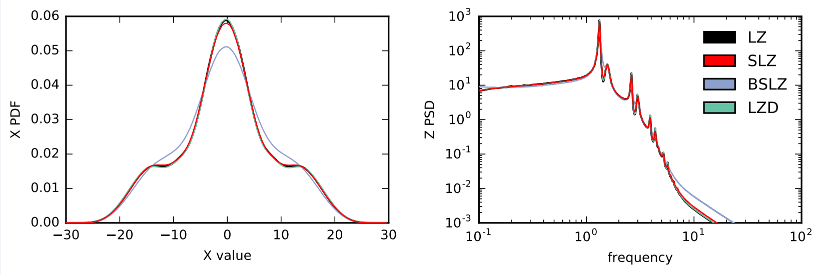
$$\frac{\mathrm{d}X}{\mathrm{d}t} = P_r(Y - X)$$

$$\mathrm{d}Y = [X(\rho - Z) - Y]\mathrm{d}t + \frac{Y}{\Upsilon^{1/2}}\mathrm{d}\mathbf{B}_t$$

$$\mathrm{d}Z = [XY - bZ]\mathrm{d}t + \frac{Z}{\Upsilon^{1/2}}\mathrm{d}\mathbf{B}_t$$

Geophysical flow modelling

Empirical PDF of X and spectrum of Z weak noise $\Upsilon = 100$



10.000 realizations $t \in [0, 100]$

BSLZ: adhoc stochastic Lorenz with multiplicative noise (Chekroun et al)

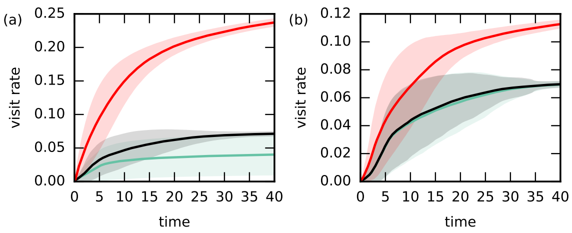
$$\frac{\mathrm{d}X}{\mathrm{d}t} = P_r(Y - X)$$

$$\mathrm{d}Y = [X(\rho - Z) - Y]\mathrm{d}t + \frac{Y}{\Upsilon^{1/2}}\mathrm{d}\mathbf{B}_t$$

$$\mathrm{d}Z = [XY - bZ]\mathrm{d}t + \frac{Z}{\Upsilon^{1/2}}\mathrm{d}\mathbf{B}_t$$

Geophysical flow modeling

Lorenz attractor visit rate $\Upsilon = 10$ and $\Upsilon = 100$



$\#\{$boxes covering the attractor visited over [0,T]$\}/$N (Computed on 10.000 realizations; red: SLZ, green: LZD, black: LZ)

Geophysical flow modelling

Usefulness of physically consistant Stochastic modeling

- Accuracy of large-scale models and error representation
- Fast exploration of attractor's regions
- **New physical modeling capabilities**

Modified wall law expression in wall bounded flows

Boundary layers and wall law

- Profile of an idealized mean velocity profile $\boldsymbol{u}(z)$ over a plane wall with constant pressure

Boundary layers stucture

- Viscous layer (dominated by molecular friction)
- Turbulent layer (dominated by large-scale shear stress associated to the unresolved fluctuations)

Two different dynamical regimes piloted by different physics
Buffer zone at the interface

Modified wall law expression in wall bounded flows

Viscous layer stationary equations

$$\text{Large-scale component} : \nu \boldsymbol{\nabla}^2 \boldsymbol{u} = 0 \Rightarrow \partial_z \boldsymbol{u} = C_1,$$

$$\text{Small scale component} : \nu \boldsymbol{\nabla}^2 \boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t = 0 \Rightarrow \boldsymbol{\nabla}^2 \boldsymbol{\sigma}^{ij} = 0,$$

$$\text{Turbulent pressure} : \mathrm{d}p_t = C_2,$$

$$\text{Incompressibility} : \boldsymbol{\nabla} \cdot (\boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t) = 0$$

$\Rightarrow$ Mean Velocity profile

$$\forall z \in [0, z_0] \;\; \boldsymbol{u}(z)\Delta t = \frac{1}{\nu} \bar{U}_\tau^2 z \, \boldsymbol{\delta u} \Delta t + \epsilon z (\Delta t)^{1/2} \boldsymbol{\eta}.$$

Small-scale component: 2D divergence free random field with variance, which depends on the wall shear stress variance and with a quadratic growth in z

Modified wall law expression in wall bounded flows

Turbulent layer stationary equations

Large-scale component : $-\partial_z a_{zz}\partial_z \boldsymbol{u} - \partial_z\big((a_{zz}+2\nu)\partial_z \boldsymbol{u}\big) = 0,$

Turbulent pressure : $\boldsymbol{\nabla}_H \mathrm{d}p_t = \partial_z \boldsymbol{u}(\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t)_z + \nu\boldsymbol{\nabla}^2(\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t)_H = 0,$

Turbulent pressure : $\partial_z \mathrm{d}p_t = \nu\boldsymbol{\nabla}^2(\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t)_z,$

Incompressibility : $\boldsymbol{\nabla}\cdot(\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t) = 0 \;\; \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\mathbf{a}) = 0$

Logarithmic law for the mean velocity profile if the turbophoresis drift is neglected and linear turbulent viscosity

Buffer sublayer

Incompressibility: $\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{a} = 0 \implies \partial^2_{zz}a = 0 \implies a_{zz}(z) = \widetilde{\kappa}\widetilde{U}_\tau(z-z_0)$

At the interface $z = z_0$, $\partial_z \boldsymbol{u}(z_0) = \frac{1}{\nu}\bar{U}_\tau^2\boldsymbol{\delta u}$

$\implies$ Mean velocity profile:

$$\boxed{\forall z\in[z_0,z_L] \;\; \boldsymbol{u}(z) = \boldsymbol{u}(z_0) - \bar{U}_\tau\frac{4\nu}{\widetilde{\kappa}}\left(\frac{1}{\widetilde{\kappa}\bar{U}_\tau(z-z_0)+2\nu} - \frac{1}{2\nu}\right)\delta\boldsymbol{u}}$$

Modified wall law expression in wall bounded flows

Turbulent layer stationary equations

Large-scale component : $-\partial_z a_{zz}\partial_z \boldsymbol{u} - \partial_z\big((a_{zz}+2\nu)\partial_z \boldsymbol{u}\big) = 0,$

Turbulent pressure : $\boldsymbol{\nabla}_H \mathrm{d}p_t = \partial_z \boldsymbol{u}(\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t)_z + \nu\boldsymbol{\nabla}^2(\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t)_H = 0,$

Turbulent pressure : $\partial_z \mathrm{d}p_t = \nu\boldsymbol{\nabla}^2(\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t)_z,$

Incompressibility : $\boldsymbol{\nabla}\cdot(\boldsymbol{\sigma}\mathrm{d}\mathbf{B}_t) = 0 \quad \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\mathbf{a}) = 0$

Logarithmic sublayer

To get a logarithmic law, $a_{zz} \sim \sqrt{z}$ and by continuity

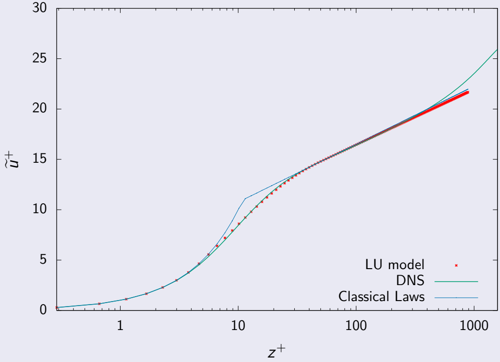
$$a_{zz}(z) = \widetilde{\kappa}\bar{U}_\tau(z_L - z_0)\sqrt{\frac{z}{z_L}}, \quad \forall z \in [z_L, z_1]$$

$$\boxed{\boldsymbol{u}(\mathrm{z}) = \boldsymbol{u}(\mathrm{z}_L) + \partial_z \boldsymbol{u}(z_L) z_L \ln\left(\frac{z}{z_L}\right)}$$

with the factor $z_L\partial_z \boldsymbol{u}(z_L) = (4\nu\bar{U}_\tau^2 z_L)/\big(\widetilde{\kappa}\bar{U}_\tau(z_L - z_0) + 2\nu\big)^2$

Geophysical flow modeling

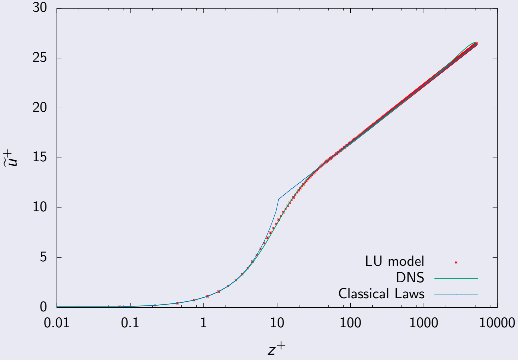
Numerical results turbulent boundary layer flow



Re_τ	z_0^+	z_L^+	$\widetilde{\kappa}$
1989	4.90	50.38	0.158

Geophysical flow modeling

Numerical results channel flow



Re_τ	z_0^+	z_L^+	$\widetilde{\kappa}$
5200	5.0	45.0	0.16

Role of small-scale inhomogeneity in flow structuration

From Euler-LU to Craik-Leibovich-LU

Euler-LU Momentum:

$$\mathrm{d}_t \boldsymbol{w} + \left((\widetilde{\boldsymbol{w}} \mathrm{d}t + \boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t) \cdot \boldsymbol{\nabla} \right) \boldsymbol{w} - \frac{1}{2} \boldsymbol{\nabla} \cdot (\boldsymbol{a} \boldsymbol{\nabla} \boldsymbol{w}) \mathrm{d}t + 2\boldsymbol{\Omega} \times \boldsymbol{w} \mathrm{d}t = -\boldsymbol{\nabla} p$$

$$\widetilde{\boldsymbol{w}} = \boldsymbol{w} - \frac{1}{2} \boldsymbol{\nabla} \cdot \boldsymbol{a} = \boldsymbol{w} + \boldsymbol{w}_S$$

Change of variable, introduction of a modified pressure, turbophoresis drift stationnary and quasi-harmonic:

$$\mathrm{d}_t \widetilde{\boldsymbol{w}} + \left((\widetilde{\boldsymbol{w}} \mathrm{d}t + \boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t) \cdot \boldsymbol{\nabla} \right) \widetilde{\boldsymbol{w}} - \overbrace{\frac{1}{2} \boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \cdot \left(\boldsymbol{a} (\widetilde{\boldsymbol{w}} - \boldsymbol{w}_S)^T \right)}^{\text{SGS}} + \overbrace{(\boldsymbol{w}_S \cdot \boldsymbol{\nabla}) \boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t}^{\text{noise advection}} = -\boldsymbol{\nabla} \pi$$
$$- \underbrace{2\boldsymbol{\Omega} \times \widetilde{\boldsymbol{w}}}_{\text{modified Coriolis}} \mathrm{d}t + \underbrace{2\boldsymbol{\Omega} \times \boldsymbol{w}_S}_{\text{Coriolis Stokes}} \mathrm{d}t - \underbrace{\boldsymbol{w}_S \times \left(\widetilde{\boldsymbol{\omega}} \mathrm{d}t + \boldsymbol{\nabla} \times (\boldsymbol{\sigma} \mathrm{d}\mathbf{B}_t) \right)}_{\text{vortex force}}$$

LES\LU deterministic form corresponds to Craik-Leibovich system

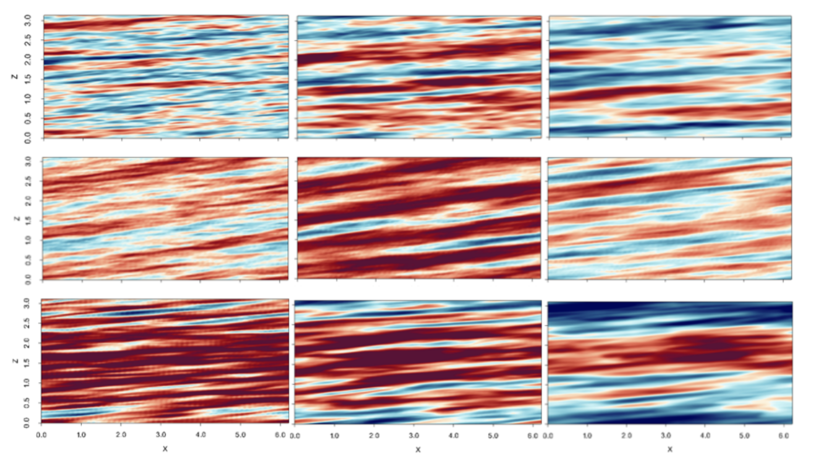
$$\partial_t \widetilde{\boldsymbol{w}} + (\widetilde{\boldsymbol{w}} \cdot \boldsymbol{\nabla}) \widetilde{\boldsymbol{w}} - \frac{1}{2} \boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \cdot \left(\boldsymbol{a} (\widetilde{\boldsymbol{w}} - \boldsymbol{w}_S)^T \right) = -\boldsymbol{\nabla} \pi - \underbrace{2\boldsymbol{\Omega} \times (\widetilde{\boldsymbol{w}} - \boldsymbol{w}_S)}_{\text{Coriolis + Stokes}} - \underbrace{\boldsymbol{w}_S \times \widetilde{\boldsymbol{\omega}}}_{\text{vortex force}}$$

with pressure term $\pi = p + \frac{1}{2} \|\boldsymbol{w} - \boldsymbol{w}_S\|^2 - \frac{1}{2} \|\widetilde{\boldsymbol{w}}\|^2$

Role of small-scale inhomogeneity in flow structuration

Numerical simulation Chanel flow $Re_\tau = 590$

Simulation of a DNS, LES 1/48 (Dynamic Smagorinsky) and LU 1/48 (Full stochastic model with noise learned from POD/EOF on DNS)

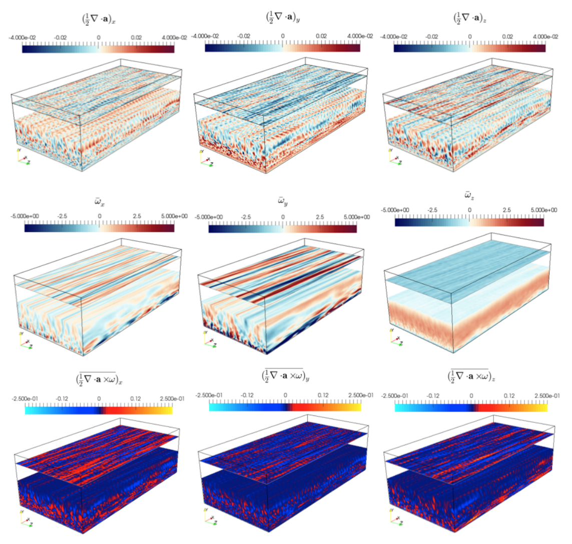


row: DNS, LU, DSM; col: $y^+ = 50$, $y^+ = 100$, $y^+ = 200$

Role of small-scale inhomogeneity in flow structuration

Numerical simulation Chanel flow $Re_\tau = 590$

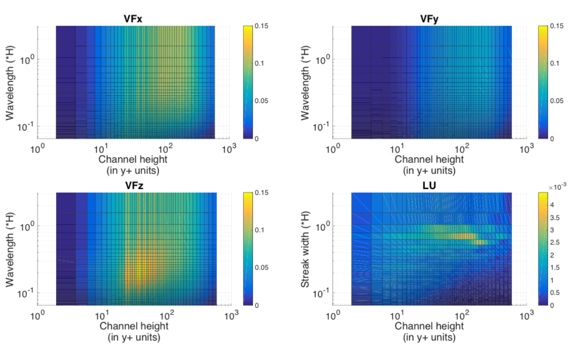
Turbophoresis drift, Vorticity, Vortex force



Role of small-scale inhomogeneity in flow structuration

Numerical simulation Chanel flow $Re_\tau = 590$

1D FFT of vortex force components



Conclusion

Dynamics modeling through stochastic transport

- LU modeling allows a systematic derivation of stochastic fluid dynamics models
- Noise brings an additional degree of freedom for modeling
- Good representation of large-scale models (more accurate and faster)
- Data analysis tool (to decipher the role of small-scale) (see also Resseguier et al JFM 2017)
- Modeling tool to explain and simulate events related to small-scale activity

Perspectives

Ocean modelling

- Derivation of Air-Sea interaction models
- Generalize the idea of modified advection and instabilities
- Study of the wave solutions(Rossby waves, internal waves,...)
- Theoretical study of the associated SPDEs
- Interaction of waves and mean large scale current
- Full stochastic simulation of realistic ocean dynamics models
- Data assimilation going toward non Gaussian models

Recent Publications

B. Pinier, E. Mémin, S. Laizet, R. Lewandowski
A stochastic flow model to predict the mean velocity in wall bounded flows, hal-inria, 2019
Y. Yang, E. Mémin
Estimation of physical parameters under location uncertainty using an Ensemble^2-Expectation-Maximization algorithm, QJRMS, 2019
B. Chapron, P. Dérian, E. Mémin, V. Resseguier Large scale flows under location uncertainty: a consistent stochastic framework, QJRMS, 2018
P. Chandramouli, D. Heitz, S. Laizet, E. Mémin Coarse large-eddy simulations in a transitional wake flow with flow models under location uncertainty, Comp. and Fluids. 2018
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