Infinite Volume Gibbs Point Process

Lecture 4

G.P.P on $\mathbb{R}^d = \Lambda$

$\longrightarrow$ spatial statistic considered

$\longrightarrow$ phase transition results

$$\ll \cancel{P^{3,\beta}_{\mathbb{R}^d} = \frac{1}{Z} e^{-\beta H} \pi^3_{\mathbb{R}^d}} \gg$$

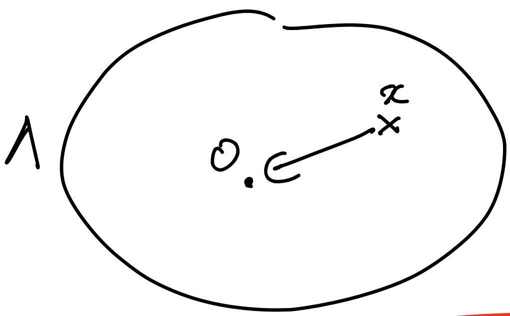
$$\mathbb{P}^{3,\beta}_{\Lambda} \xrightarrow[\Lambda \to \mathbb{R}^d]{?} \mathbb{P}^{3,\beta} ?$$

I) Existence of infinite volume G.P.P.

$$\boxed{\begin{array}{l} \text{Definition} \quad (P_n) \xrightarrow[n\to+\infty]{\mathcal{L}\mathcal{L}} P \leftarrow \text{Local convergence} \\ \text{if for any local and tame function } f \\ \text{(i.e. } f(\gamma) = f(\gamma_{\Lambda}) \text{ and } |f(\gamma_\Lambda)| \leqslant C \,\#\gamma_\Lambda) \\ \qquad\qquad \text{bounded} \\ \displaystyle\int f \, dP_n \xrightarrow[n\to+\infty]{} \int f \, dP \end{array}}$$

$$\overline{P^{z,\beta}_\Lambda} = \frac{1}{|\Lambda|}\int_\Lambda P^{z,\beta}_\Lambda \circ \tau_x^{-1}\,dx.$$

translat by x.



Theorem (Georgii) [Tightness]

. $(P_\Lambda)_{\Lambda\subset\mathbb{R}^d}$ a familly of probability measures.

If there exists a constant $C \geq 0$ s.t.

$\forall \Lambda$ $\quad I(P_\Lambda \mid \pi^z_\Lambda) \leq C\left(|\Lambda| + E_{P_\Lambda}(N_\Lambda)\right)$

Then, $\quad (\overline{P_\Lambda})$ is tight for the local topology

corollary: $\quad \left(\overline{P^{z,\beta}_\Lambda}\right)$ is tight.

Any accumulation point is called _infinite Volume GPP_

$\overline{P}^{3,\beta}$ accumulod point

$\overline{P}^{3,\beta}$?

Assumptions: $\longrightarrow$ Ruelle superstability

$\longrightarrow$ Geometric interod (Jeagui Der Droeuilhet)

$\longrightarrow$ $\boxed{\text{Finite Range.}}$

Assumption: there exists $R \geq 0$

$\forall x, \forall \gamma \quad h(x,\gamma) = h(x, \gamma_{B(x,R)})$

$\boxed{\text{Finite Range ass.}}$

Any Accumulation point $\mathbb{P}^{3,\beta}$ satisfies the GNZ equod

Definition any Probability measure satisfying the GNZ equations is called infinite Gibbs Point Process.

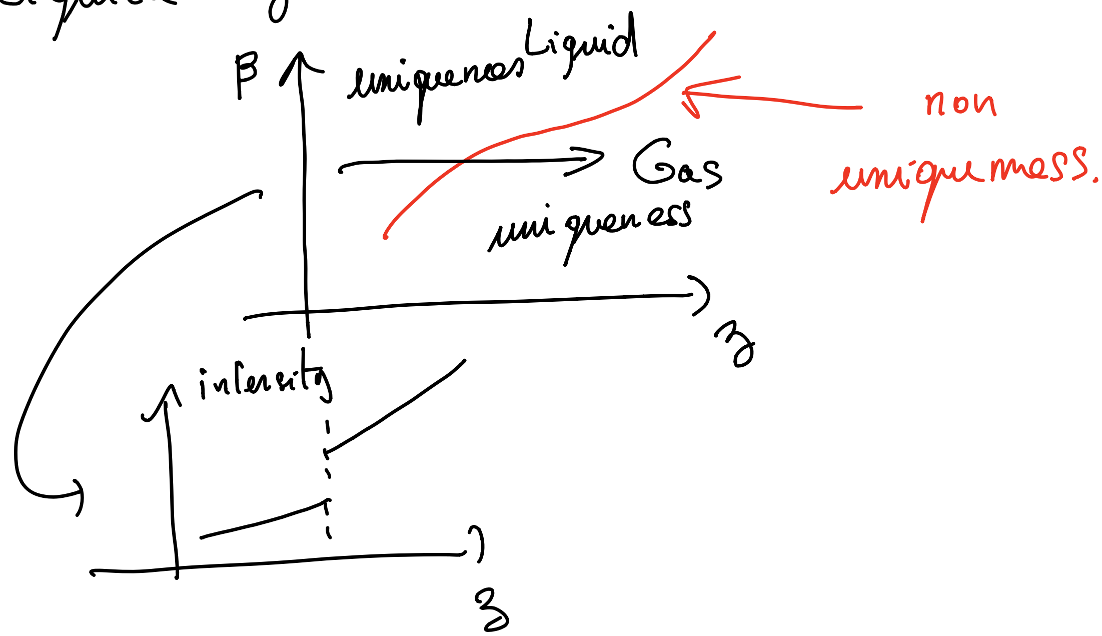
any I.G.P.P is an accumulation Point for

$$\left(P^{z,\beta}_{\Lambda},\ \text{boundary condition}\right)_{\Lambda}$$

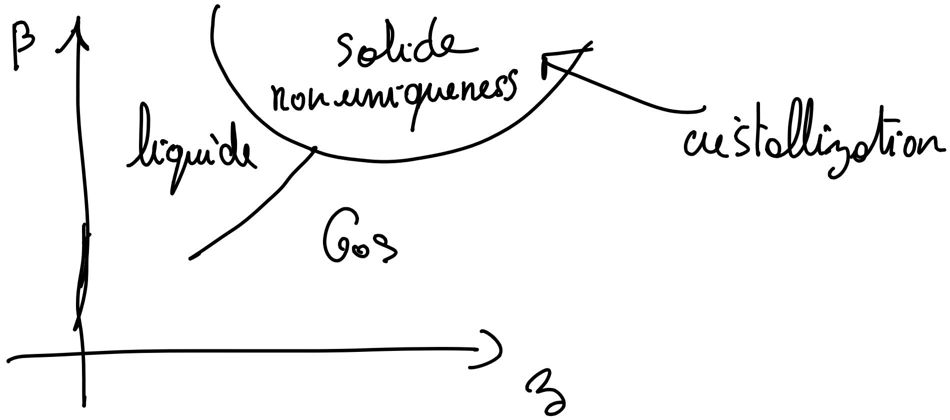
extremal points are ergodic. (Prediction)

II) Phase transition problem

Liquid - Gas Phase transition



Liquide – Solide Phase transition

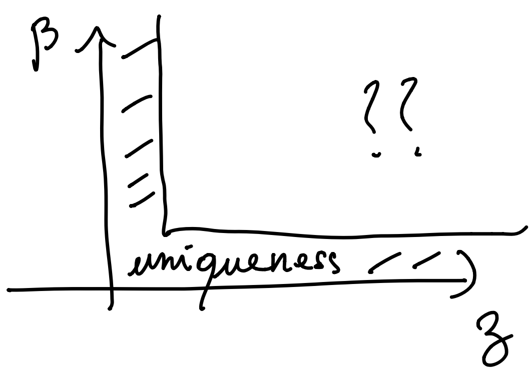


uniqueness: . Kirkwood Salsburg equation
(Ruelle 70)

. disagreement Percolation
(Houdebert – Temmel 17)

. cluster expansion.

$z \ll 1$ or $\beta \ll 1 \implies$ uniqueness

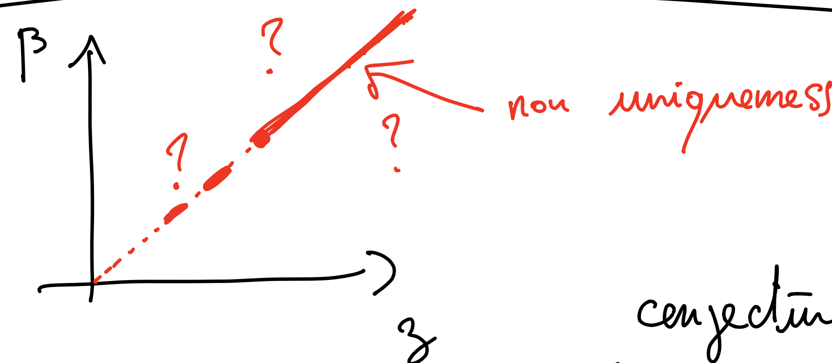


Area-Interaction Widom Rowlison 70.

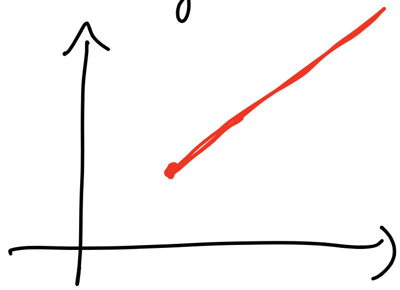
$$H(\gamma) = \text{Volume}\Big(\bigcup_{x\in\gamma} B(x,R)\Big)$$

Theorem Chayes - Chayes - Kotecky. 95

non uniqueness if $z=\beta$ large enough.

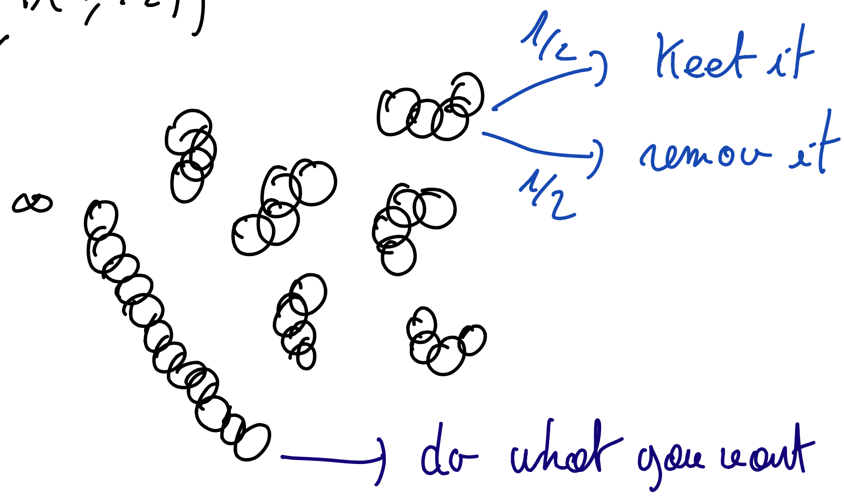


conjecture



$$\tilde{H} = -N_{cc}\Big(\bigcup_{x\in\gamma} B(x,R/2)\Big)$$

$$\tilde{\beta} = \ln(2)$$

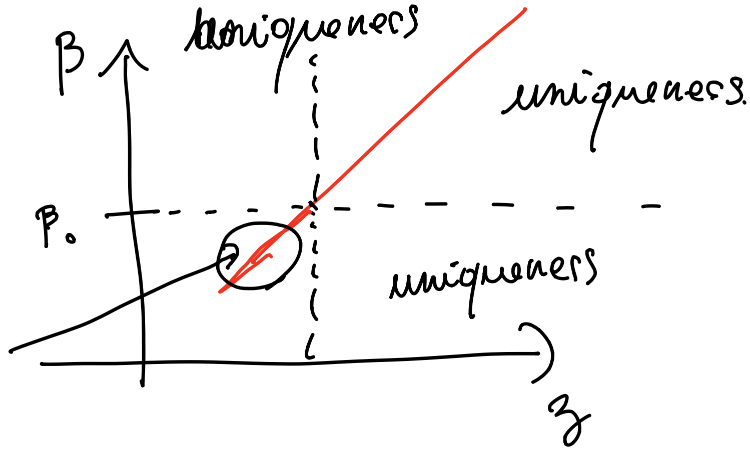


Percolat $\Longrightarrow$ non uniqueness

Theorem Der. Houdebert 2018

$\exists \beta_0 \geq 0$ s.t $\forall \beta \geq \beta_0$

non uniqueness $\iff$ $z = \beta$



Toubles