Multi-stage sequential sampling models: A framework for binary choice options Part 4

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Overview

- Part 1
  - Motivation – 3 Examples
  - Basic assumptions of sequential sampling models (as used here)
  - Multi-stage sequential sampling models
- Part 2
  - Time and order schedules
  - Implementation
  - Predictions
  - Impact of attention time distribution
  - Impact of attribute order
- Part 3 and 4
  - Applications

Example 2, revisited

Frames, Biases, and Rational Decision-Making in the Human Brain

Benedetto De Martino, **Dharshan Kumaran**, **Ben Seymour**, and **Raymond J. Dolan**

Abstract

Human choices are remarkably susceptible to the manner in which options are presented. This so-called "framing effect" represents a striking violation of standard economic accounts of human rationality, although its underlying neurobiology is not understood. We found that the framing effect was specifically associated with amygdala activity, suggesting a key role for an emotional system in mediating decision biases. Moreover, across individuals, orbital and medial prefrontal cortex activity predicted a reduced susceptibility to the framing effect. This finding highlights the importance of incorporating emotional processes within models of human choice and suggests how the brain may modulate the effect of these biasing influences to approximate rationality.

Science, 2006

Framing effect

Kahneman & Tversky, 1979
Tversky & Kahneman, 1981

Framing effect: cognitive bias, in which people react to a particular choice in different ways depending on how it is presented; e.g. as a loss or as a gain.

- Preference reversal
- Shift in preference

(cf. externality, description-invariance)

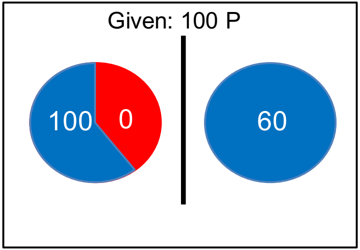
Risky choice framing

- Choice between two options
- Lotteries
- Options A is typically risk less
- Option B is risky

- **Situation 1** Outcomes are framed as **gains** (positive frame)
- **Situation 2** Outcomes are framed as **losses** (negative frame)

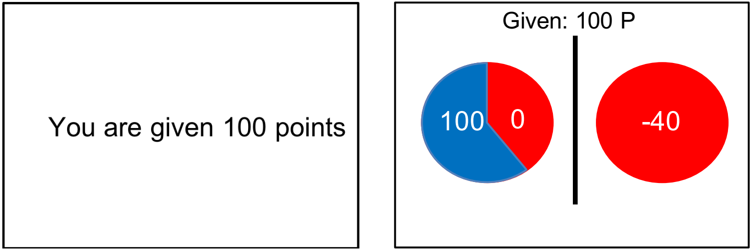
Gain frame

You are given 100 points



Keep

Loss frame



Lose

Dual process models

Applied to cognitive processes including reasoning and judgments

J.St.B.T. Evans (2008)

References	System 1	System 2
Fodor (1983, 2001)	Input modules	Higher cognition
Schneider & Schiffrin (1977)	Automatic	Controlled
Epstein (1994), Epstein & Pacini (1999)	Experiential	Rational
Chaiken (1980), Chen & Chaiken (1999)	Heuristic	Systematic
Reber (1993), Evans & Over (1996)	Implicit/tacit	Explicit
Evans (1989, 2006)	Heuristic	Analytic
Sloman (1996), Smith & DeCoster (2000)	Associative	Rule based
Hammond (1996)	Intuitive	Analytic
Stanovich (1999, 2004)	System 1 (TASS)	System 2 (Analytic)
Nisbett et al. (2001)	Holistic	Analytic
Wilson (2002)	Adaptive unconscious	Conscious
Lieberman (2003)	Reflexive	Reflective
Toates (2006)	Stimulus bound	Higher order
Strack & Deustch (2004)	Impulsive	Reflective

Dual process models

- **System 1** Intuitive (fast, emotional, biased response, . . .)
- **System 2** Deliberate (slow, rational, normative response . . .)

Most popular since Kahneman (2011), Thinking, fast and slow

Dual process models

Problems for most approaches:

- Verbal – allows no quantitative predictions
- Unclear about processing
- Reverse inference

For the few **formal** models (Loewenstein et al. 2011, Mukherjee, 2010):

- No time mechanism
- (Unclear about processing)

Basic assumptions

- Consequences of choosing each option are compared continuously over time $\rightarrow$ preferences are constructed
- Preference accumulation process with preference update
- Random fluctuation in accumulating preference strength

Preference Process

Preference strength is updated from one moment, t, to the next, $(t + h)$ by an reflecting the momentary comparison of consequences produced by imagining the choice of either option G or S with

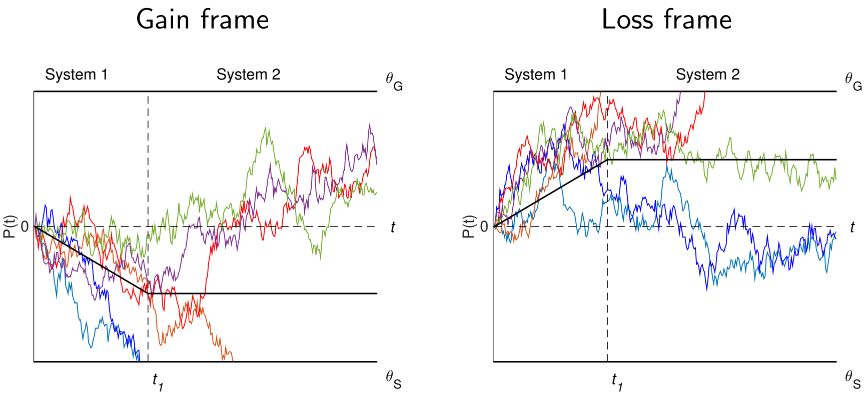
$$P(t + h) = P(t) + V_i(t + h),$$

- $V(t)$: input valence
- $V(t) = V^G(t) - V^S(t)$
- $V^G(t)$: momentary valence for the gamble
- $V^S(t)$: momentary valence for the sure option

For the example

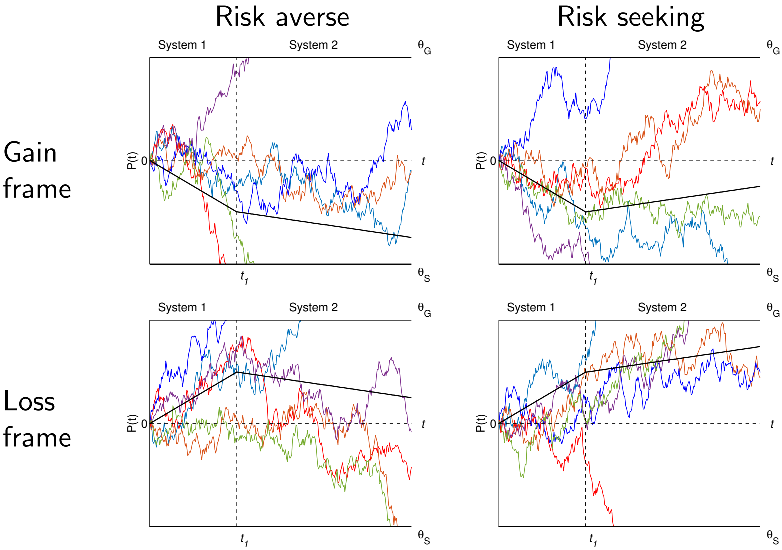
- 6 trials
- E(Sure) $=$ E(Gamble), risk neutral
- E(Sure) $\neq$ E(Gamble), risk attitudes

Gain and Loss frame



- Attention switches from system 1 to system 2 at time t_1
- Switching time may be deterministic or random (according to distribution)
- Solid lines indicate the **drift rates**

Risk attitudes

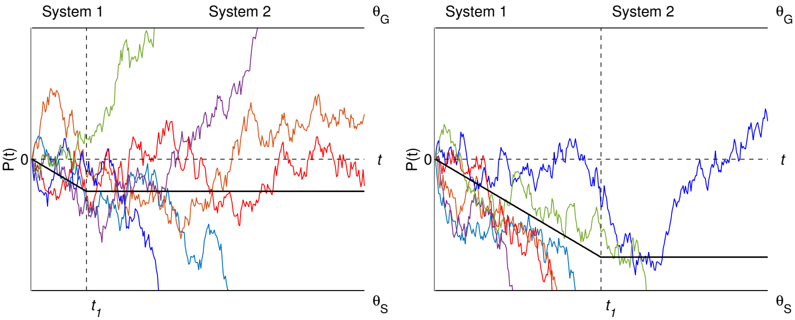


Predictions

- **Prediction 1**: The size of the framing effect is a function of the time the DM operates in System 1.
- **Prediction 2**: The size of the framing effect is a function of the time (limit) the DM has for making a choice.

Prediction 1: Attention times

The size of the framing effect is a function of the time the DM operates in System 1.

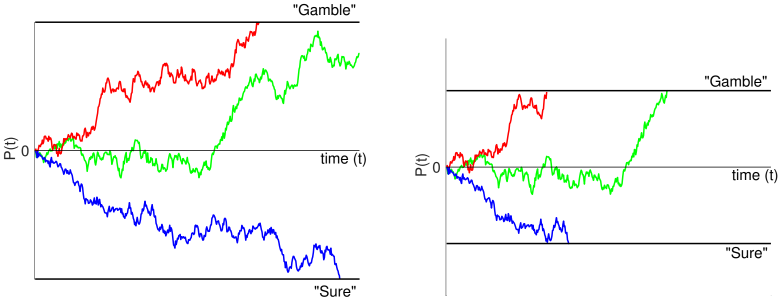


Gain frame

System 1: drift rate $< 0 \rightarrow$ Sure

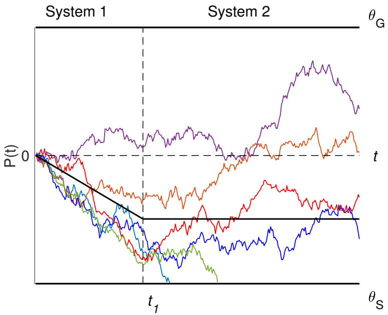
System 2: drift rate $= 0 \rightarrow$ indifferent between Gamble and Sure

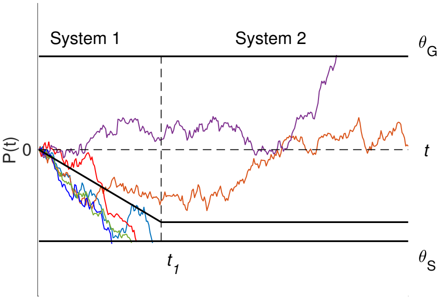
Basic assumptions – Time limit



Prediction 2: Deadlines

The size of the framing effect is a function of the time (limit) the DM has for making a choice.





Gain frame

System 1: drift rate $< 0 \rightarrow$ Sure

System 2: drift rate $= 0 \rightarrow$ indifferent between Gamble and Sure

Modeling the drift rates – Example 1

- **System 1**
  Preferences in System 1 are constructed according to **prospect theory**
- **System 2**
  Preferences in System 2 are constructed according to **expected utility theory**

System 1: Prospect Theory

- The value $\mathcal{V}$ of a simple prospect that pays x (here the starting amount) with probability p (and nothing otherwise) is given by:

$$\mathcal{V}(x, p) = w(p)v(x)$$

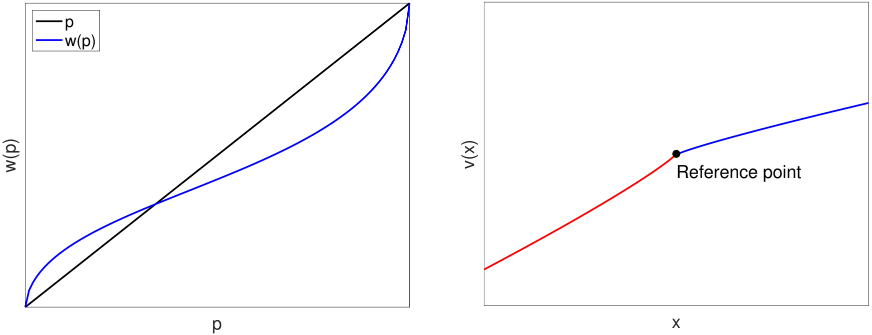
with probability weighting function

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

and value function

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda|x|^\beta & \text{if } x < 0 \end{cases}$$

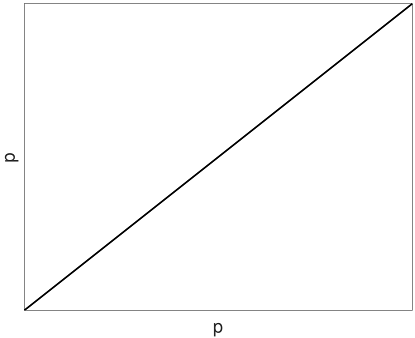
Weighting function for p Value function for x



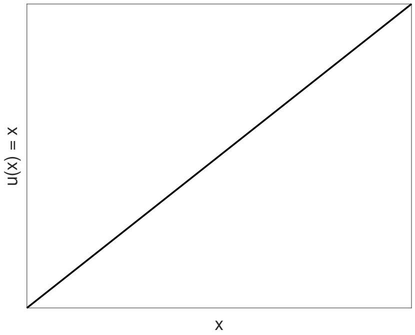
$$\begin{aligned}\mathcal{V}_G &= w(p)v_G(x)\\ \mathcal{V}_{S_{gain}} &= w(p)v_{S_{gain}}(x)\\ \mathcal{V}_{S_{loss}} &= w(p)v_{S_{loss}}(x)\end{aligned}$$

System 2: Expected Utility Theorie

Probability



Utility



p

$u(x) = x$

$$EU(x, p) = p \cdot u(x) = p \cdot x$$

Mean difference in valence (drift rate)

- System 1 – gain frame

$$\mu_{1_{gain}} = \mathcal{V}_G - \mathcal{V}_{S_{gain}}$$

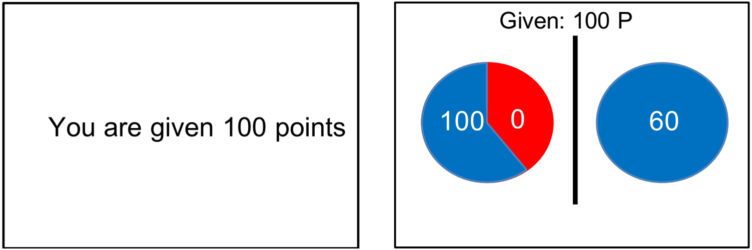
- System 1 – loss frame

$$\mu_{1_{loss}} = \mathcal{V}_G - \mathcal{V}_{S_{loss}}$$

- System 2

$$\mu_2 = EU(G) - EU(S) = 0$$

For all predictions



Amount given (reference point) r: 25, 50, 75, 100
Probability of keeping r: 0.2, 0.4, 0.6, 0.8

Predictions – Example 1

Parameters (PT from Tversky & Kahneman, 1992)

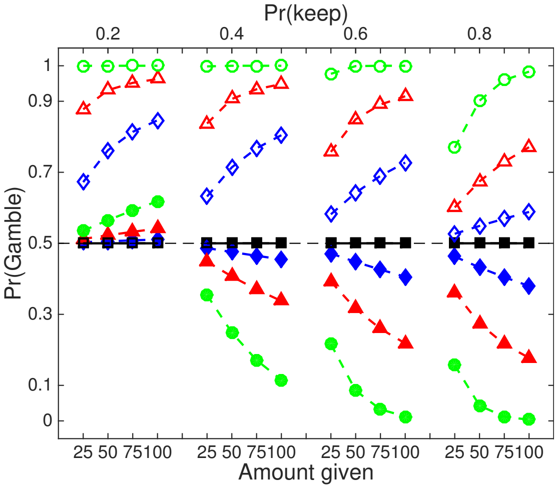
System 1	System 2
$\alpha = .88$	no parameters
$\beta = .88$	
$\lambda = 2.25$	
$\gamma = .61$	

boundary: θ

attention time: t, $E(T_1)$

Predictions 1: The size of the framing effect is a function of the time the DM operates in System 1.

$\theta = 15$;
$t_1 = 0$, 100, 500, ∞

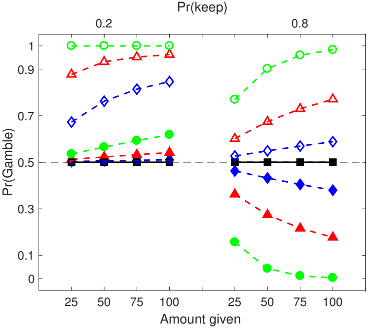


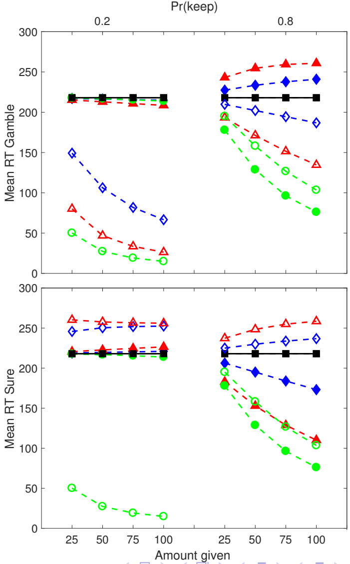
Loss frame (open)

Gain frame (filled)

Predictions 1: Attention times

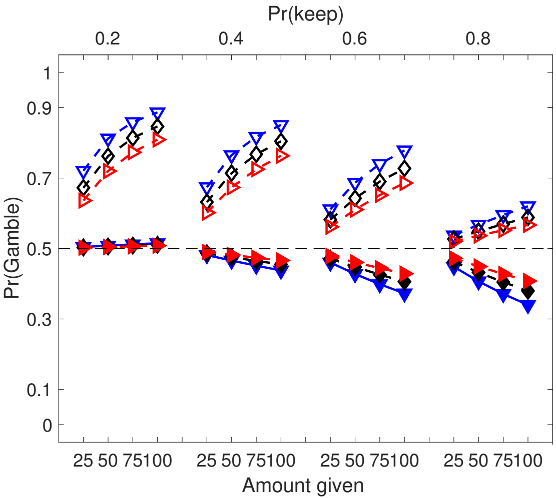
$\theta = 15$;
$t_1 = 0$, 100, 500, ∞
open: loss; filled: gain





Predictions 2: The size of the framing effect is a function of the time (limit) the DM has for making a choice.

$t_1 = 100$;
$\theta = 10, 15, 20$

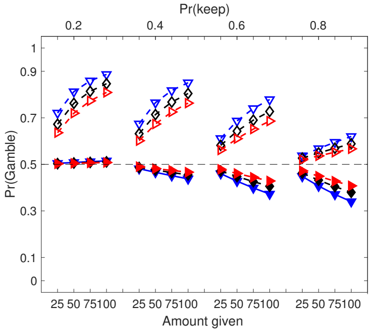


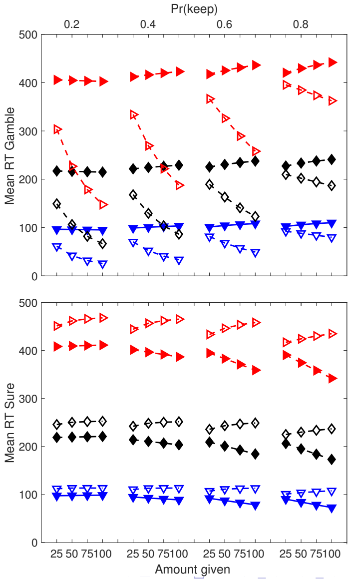
Loss frame (open)

Gain frame (filled)

Predictions 2: Time limits

$t_1 = 100$;
$\theta = 10, 15, 20$
open: loss; filled: gain





Modeling the drift rates – Example 2

- **System 1**
  Preferences in System 1 follow PT.
- **System 2**
  Preferences in System 2 are a weighted average of PT and EU.

System 2: Weighted average of PT and EU

$$\begin{aligned}\delta_2^* &= w \cdot (\mathcal{V}_G - \mathcal{V}_S) + (1 - w) \cdot (EV(G) - EV(S)) \\ &= w \cdot \delta_1 + (1 - w) \cdot \delta_2.\end{aligned}$$

- Qualitative predictions remain as before.

Modeling the drift rates – Example 3

- **System 1**
  Preferences in System 1 are modeled according to a Motivational function weighted by Willpower strength and Cognitive demand (MWC). (Loewenstein et al.,2015)
- **System 2**
  Preferences in System 2 are modeled according to EU.

System 1: MWC

- Motivational function $M(x, a)$; a captures the intensity of affective motivations
- Function $h(W, \sigma)$ reflects the willpower strength W and cognitive demands σ.

MWC

- $M(x,a) = \sum w(p_i)v(x_i,a)$
  - $w(p)$ is a probability-weighting function

$$w(p) = c + bp \text{ with } w(0) = 1, w(1) = 1, \text{ and } 0 < c < 1 - b$$

  - $v(x,a)$ is a value function that incorporates loss aversion

$$v(x,a) = \begin{cases} a\,u(x) & \text{if } x \geq 0 \\ a\lambda\,u(x) & \text{if } x < 0 \end{cases}$$

- $h(W,\sigma)$ is not specified but meant to be decreasing in W and increasing in σ.
-

$$\mathbf{V}(x) = \sum u(x_i) + h(W,\sigma) \cdot \sum w(p_i)v(x_i,a)$$

Note

WMC: static-deterministic model

For predicting choice probabilities and choice responses time

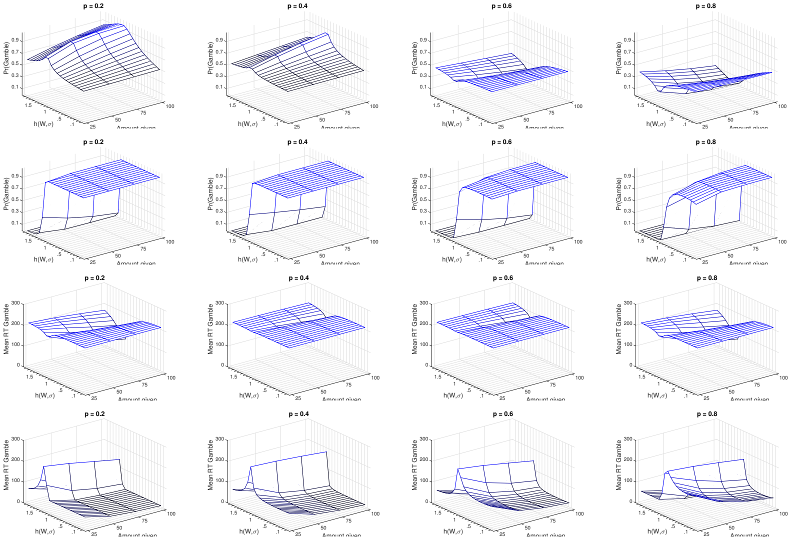
$\rightarrow$ dynamic-stochastic framework

System 1 and System 2

- MWC model assumes that both processes operate simultaneously.
- Therefore, System 1 and System 2 merge into a single drift rate and the two stages basically collapse into one single stochastic process.
- With $\mathbf{V}_G$ and $\mathbf{V}_S$ indicating the subjective value of the gamble and the sure option, respectively, the mean difference in valences (drift rates) in a gain and loss frame become

$$\begin{aligned} \delta_{gain} &= \mathbf{V}_G - \mathbf{V}_{S_{gain}} \\ \delta_{loss} &= \mathbf{V}_G - \mathbf{V}_{S_{loss}}, \end{aligned}$$

Predictions



Experiment

Guo, Trueblood, Diederich (2017), Psychological Science

- 2 $\times$ (time limits: no, 1 sec) $\times$ 2 (frames: gain, loss)
- 72 gambles per condition, collapsed to 9 "gambles" per condition
- 8 catch trials per condition
- 195 participants

Models tested

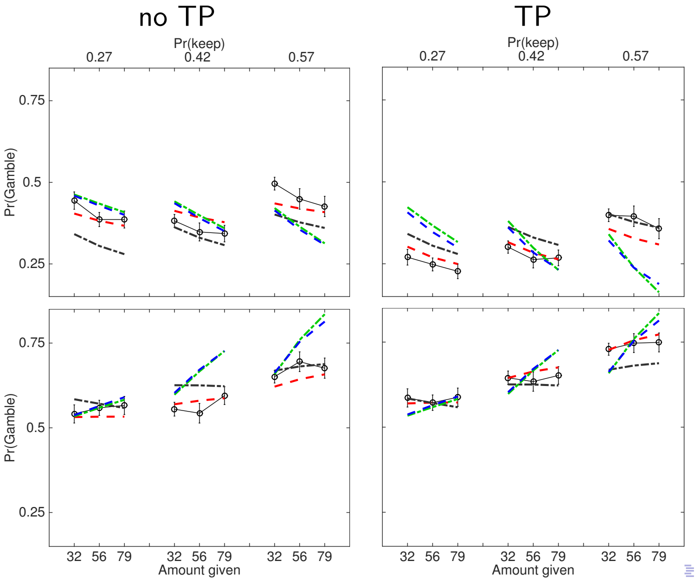
Model	Number of parameters	RMSEA
PT_k	8	1.43
PT with additional scaling factor	9	.44
Dual with PT and EU	10	.282
Dual with PT and weighted PT and EU	11	.283
MWC_k	10	1.40
MWC with additional scaling factor	11	.54
$MWC_{2stages}$	12	.49

Model acccounts: Probabilities

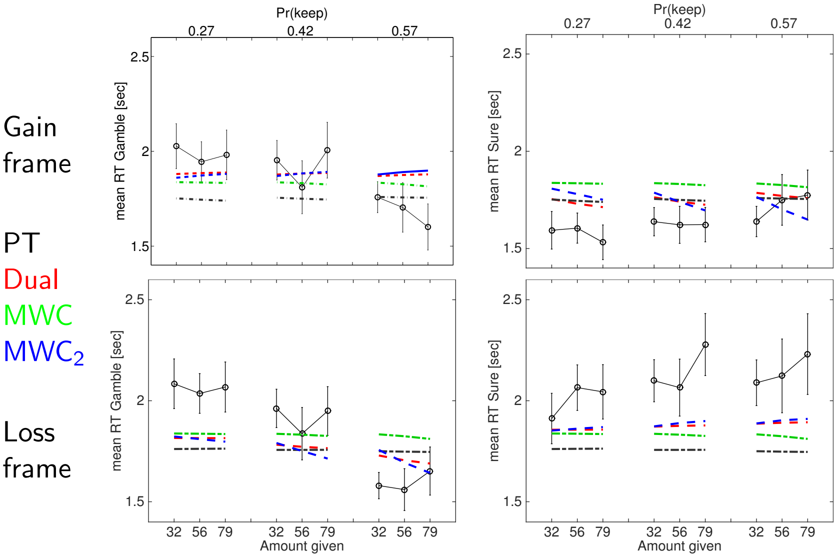
Gain frame

PT
Dual
MWC
MWC$_2$

Loss frame



Model acccounts: RT – no TP

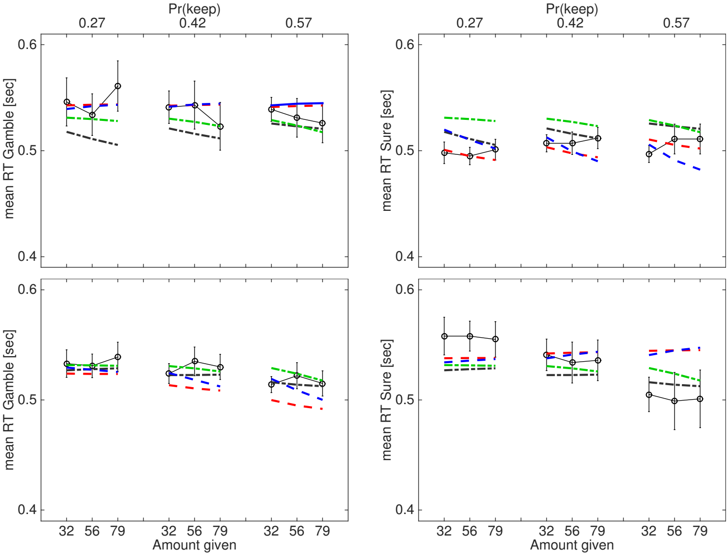


Model acccounts: RT – TP

Gain frame

PT
Dual
MWC
MWC_2

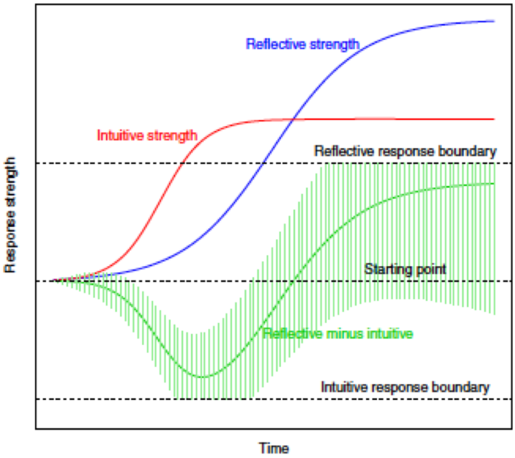
Loss frame



Further directions

Baron & Gürcay (2017) for moral judgments

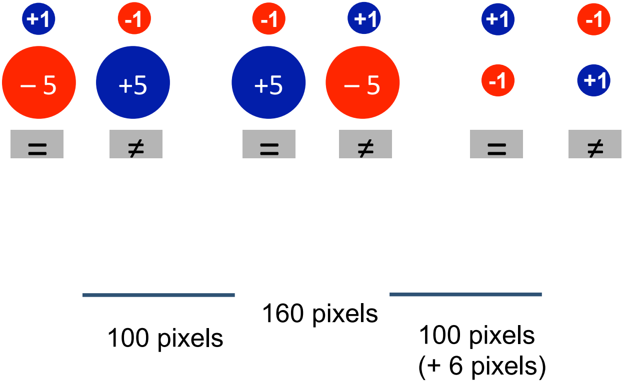
$RT = b_0 + b_1 AD + b_2 U + b_3 AD \cdot U$



Example 1, revisited

Influence of payoffs and discrimination with manipulated processing orders

Payoffs and discrimination

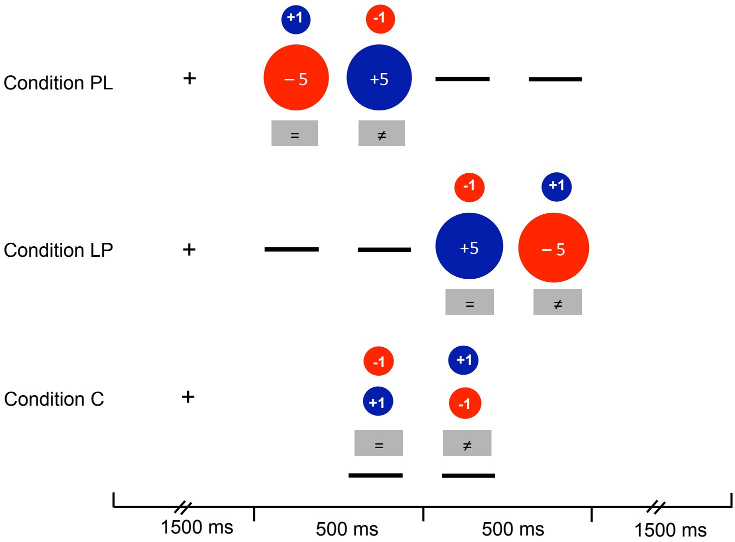


Diederich & Busemeyer (2006); Diederich (2008), with time constraints

Attributes and task

- Attribute 1: Payoffs (Same, Different, Neutral)
- Attribute 2: Lines (*same*, *different*)
- Task: "same"/"different" judgment

Presentation orders



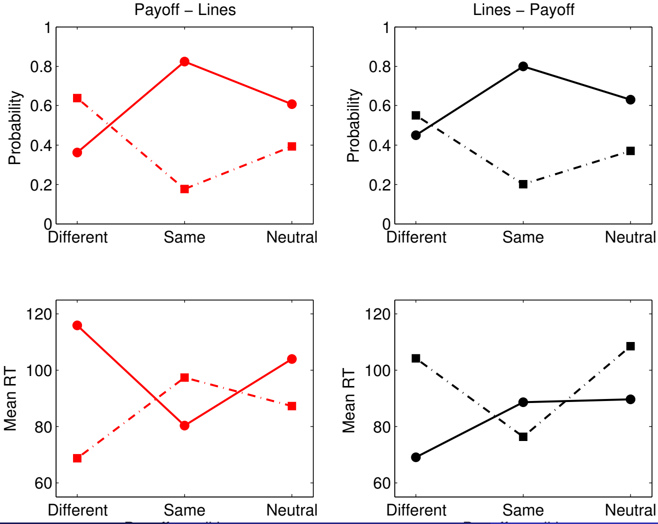
All conditions

Lines stimulus	Payoff matrix	Presentation order
different	Different (**D**)	Payoff–Lines (PL)
same	Same (**S**)	Lines–Payoff (LP)
	Neutral (**N**)	Payoff/Lines (C)

$2 \times 3 \times 3$ factorial design

Predictions for Payoff–Line example

$\mu_1 = .1, -.1, 0$ for payoffs D, S, N; $\mu_2 = -.05$ for line *same*



Model specification

- **Order schedule**: The attribute sequence is either $(1, 2, 1, 2, \ldots)$ or $(2, 1, 2, 1, \ldots)$, depending on whether $k_1 = 1$ or $k_1 = 2$
- **Time schedule**: Geometric distribution
  $Pr(T = n) = (1 - r)^{n-1} r, \;\; n = 1, 2, \ldots$
  Expected value: $E(T) = 1/r$

Payoffs and Lines: Parameter estimation

Two stages

- Parameter estimated simultaneously for conditions PL and LP
- Cross validation for condition C (some of the parameters same as for PL and LP)

Payoffs and Lines: Models tested

- M1: Payoffs processed first and then switch to the lines (PL and C).
- M2: Lines processed first and then switch to the payoffs (LP and C).
- M3: Payoffs processed first, switch to the lines and then switch back and forth between attributes (C).
- M4: Lines processed first, switch to the payoffs and then switch back and forth between attributes (C).
- M5: Start with any attribute and switch back and forth between them (C).

Payoffs and Lines: Parameters estimated for PL and LP

Payoffs	μ_{PD}, μ_{PS}
Lines	μ_{Ls}, μ_{Ld}
Attention switching	r_{12}, r_{21}
Decision bound	θ_{PL}, θ_{LP}
Residual	R_{PL}, R_{LP}

from 36 data points

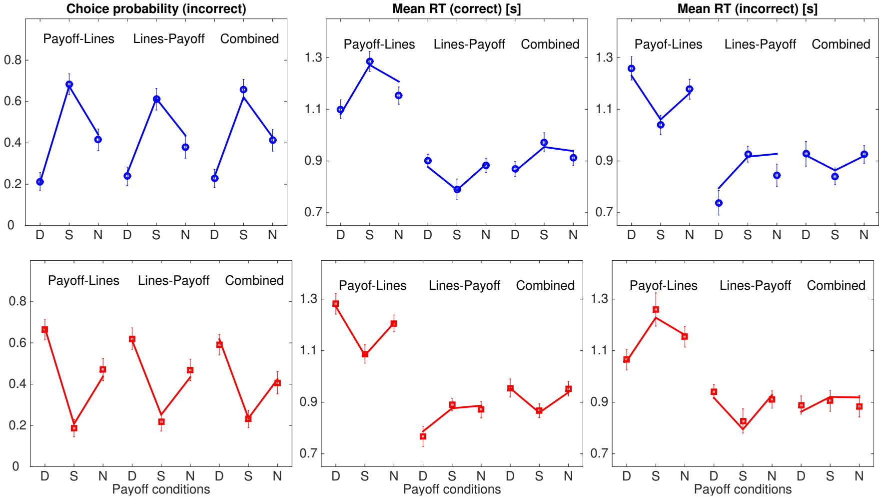
Payoffs and Lines: Parameters estimated for C

Cross validation

Attention switching	r_{12} or/and r_{21}
Decision bound	θ_C
Residual	R_C

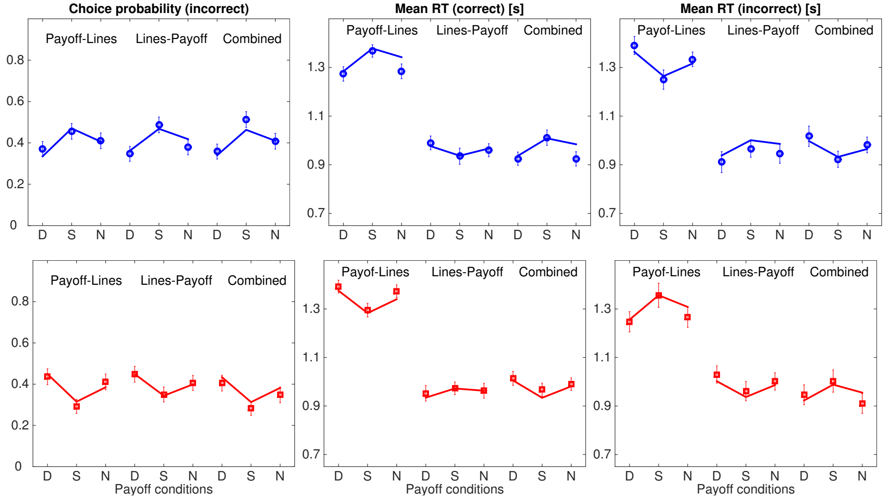
from 18 data points

Lines *different* *same*



Results – Group 3

Lines *different* *same*



Best fit

For all groups:

- M3: Payoffs processed first, switch to the lines and then switch back and forth between attributes (C).

Response patterns

Group	Stimuli	Presentation					
		PL		LP		C	
		obs	pred	obs	pred	obs	pred
	PD Ld	f	+	s	+	f	+
	PS Ld	f	+	s	+	f	+
1	PN Ld	f	−	s	−	f	−
	PD Ls	f	+	s	+	f	+
	PS Ls	f	+	s	+	f	+
	PN Ls	s	+	f	+	s	+
	PD Ld	f	+	s	+	f	+
	PS Ld	s	+	f	+	f	−
3	PN Ld	f	−	s	−	f	−
	PD Ls	s	+	f	+	s	+
	PS Ls	f	+	s	+	f	+
	PN Ls	s	+	f	+	s	+

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