Multi-stage sequential sampling models: A framework for binary choice options
Part 4

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March 18 – 22, 2019
Overview

Part 1
- Motivation – 3 Examples
- Basic assumptions of sequential sampling models (as used here)
- Multi-stage sequential sampling models

Part 2
- Time and order schedules
- Implementation
- Predictions
- Impact of attention time distribution
- Impact of attribute order

Part 3 and 4
- Applications
Frames, Biases, and Rational Decision-Making in the Human Brain

Benedetto De Martino, Dharshan Kumaran, Ben Seymour, and Raymond J. Dolan

Abstract

Human choices are remarkably susceptible to the manner in which options are presented. This so-called “framing effect” represents a striking violation of standard economic accounts of human rationality, although its underlying neurobiology is not understood. We found that the framing effect was specifically associated with amygdala activity, suggesting a key role for an emotional system in mediating decision biases. Moreover, across individuals, orbital and medial prefrontal cortex activity predicted a reduced susceptibility to the framing effect. This finding highlights the importance of incorporating emotional processes within models of human choice and suggests how the brain may modulate the effect of these biasing influences to approximate rationality.

Science, 2006
Framing effect

Kahneman & Tversky, 1979
Tversky & Kahneman, 1981

**Framing effect**: cognitive bias, in which people react to a particular choice in different ways depending on how it is presented; e.g. as a loss or as a gain.

- Preference reversal
- Shift in preference

(cf. externality, description-invariance)
Risky choice framing

- Choice between two options
- Lotteries
- Options A is typically risk less
- Option B is risky

**Situation 1** Outcomes are framed as *gains* (positive frame)

**Situation 2** Outcomes are framed as *losses* (negative frame)
Gain frame

You are given 100 points

Given: 100 P

100

60

Keep
You are given 100 points

Given: 100 P

Lose

-40
Dual process models

Applied to cognitive processes including reasoning and judgments

J.St.B.T. Evans (2008)

<table>
<thead>
<tr>
<th>References</th>
<th>System 1</th>
<th>System 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fodor (1983, 2001)</td>
<td>Input modules</td>
<td>Higher cognition</td>
</tr>
<tr>
<td>Schneider &amp; Schiffrin (1977)</td>
<td>Automatic</td>
<td>Controlled</td>
</tr>
<tr>
<td>Hammond (1996)</td>
<td>Intuitive</td>
<td>Analytic</td>
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<tr>
<td>Nisbett et al. (2001)</td>
<td>Holistic</td>
<td>Analytic</td>
</tr>
<tr>
<td>Lieberman (2003)</td>
<td>Reflexive</td>
<td>Reflective</td>
</tr>
<tr>
<td>Toates (2006)</td>
<td>Stimulus bound</td>
<td>Higher order</td>
</tr>
<tr>
<td>Strack &amp; Deustch (2004)</td>
<td>Impulsive</td>
<td>Reflective</td>
</tr>
</tbody>
</table>
Dual process models

- **System 1** Intuitive (fast, emotional, biased response, ...)
- **System 2** Deliberate (slow, rational, normative response ...)

Most popular since Kahneman (2011), Thinking, fast and slow
**Problems** for most approaches:

- Verbal – allows no quantitative predictions
- Unclear about processing
- Reverse inference

For the few **formal** models (Loewenstein et al. 2011, Mukherjee, 2010):

- No time mechanism
- (Unclear about processing)
Basic assumptions

- Consequences of choosing each option are compared continuously over time → preferences are constructed
- Preference accumulation process with preference update
- Random fluctuation in accumulating preference strength
Preference Process

Preference strength is updated from one moment, $t$, to the next, $(t + h)$ by an reflecting the momentary comparison of consequences produced by imagining the choice of either option $G$ or $S$ with

$$P(t + h) = P(t) + V_i(t + h),$$

- $V(t)$: input valence
- $V(t) = V^G(t) - V^S(t)$
- $V^G(t)$: momentary valence for the gamble
- $V^S(t)$: momentary valence for the sure option
For the example

- 6 trials
- $E(\text{Sure}) = E(\text{Gamble})$, risk neutral
- $E(\text{Sure}) \neq E(\text{Gamble})$, risk attitudes
Attention switches from system 1 to system 2 at time $t_1$

Switching time may be deterministic or random (according to distribution)

Solid lines indicate the drift rates
Risk attitudes

Risk averse

Risk seeking

Gain frame

Loss frame
• **Prediction 1**: The size of the framing effect is a function of the time the DM operates in System 1.

• **Prediction 2**: The size of the framing effect is a function of the time (limit) the DM has for making a choice.
Prediction 1: Attention times

The size of the framing effect is a function of the time the DM operates in System 1.

Gain frame
System 1: drift rate < 0 → Sure
System 2: drift rate = 0 → indifferent between Gamble and Sure
Basic assumptions – Time limit

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Multi-stage models

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Prediction 2: Deadlines

The size of the framing effect is a function of the time (limit) the DM has for making a choice.

Gain frame

System 1: drift rate $< 0 \rightarrow$ Sure
System 2: drift rate $= 0 \rightarrow$ indifferent between Gamble and Sure
System 1
Preferences in System 1 are constructed according to **prospect theory**

System 2
Preferences in System 2 are constructed according to **expected utility theory**
The value $V$ of a simple prospect that pays $x$ (here the starting amount) with probability $p$ (and nothing otherwise) is given by:

$$V(x, p) = w(p)v(x)$$

with probability weighting function

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1 - p)^\gamma)^{1/\gamma}}$$

and value function

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda|x|^\beta & \text{if } x < 0 \end{cases}$$
System 1: Prospect Theory

**Weighting function for $p$**

$$w(p)$$

**Value function for $x$**

$$v(x)$$

Reference point

$$V_G = w(p)v_G(x)$$

$$V_{S_{gain}} = w(p)v_{S_{gain}}(x)$$

$$V_{S_{loss}} = w(p)v_{S_{loss}}(x)$$
System 2: Expected Utility Theory

Probability

Utility

\[ p \]

\[ u(x) = x \]

\[ EU(x, p) = p \cdot u(x) = p \cdot x \]
Mean difference in valence (drift rate)

- **System 1 – gain frame**
  \[
  \mu_{1_{\text{gain}}} = \mathcal{V}_G - \mathcal{V}_{S_{\text{gain}}}
  \]

- **System 1 – loss frame**
  \[
  \mu_{1_{\text{loss}}} = \mathcal{V}_G - \mathcal{V}_{S_{\text{loss}}}
  \]

- **System 2**
  \[
  \mu_2 = EU(G) - EU(S) = 0
  \]
For all predictions

You are given 100 points

Amount given (reference point) $r$: 25, 50, 75, 100
Probability of keeping $r$: 0.2, 0.4, 0.6, 0.8
Parameters (PT from Tversky & Kahneman, 1992)

<table>
<thead>
<tr>
<th>System 1</th>
<th>System 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = .88$</td>
<td>no parameters</td>
</tr>
<tr>
<td>$\beta = .88$</td>
<td></td>
</tr>
<tr>
<td>$\lambda = 2.25$</td>
<td></td>
</tr>
<tr>
<td>$\gamma = .61$</td>
<td></td>
</tr>
<tr>
<td>boundary: $\theta$</td>
<td></td>
</tr>
<tr>
<td>attention time: $t$, $E(T_1)$</td>
<td></td>
</tr>
</tbody>
</table>
Predictions 1: The size of the framing effect is a function of the time the DM operates in System 1.

\[ \theta = 15; \]
\[ t_1 = 0, 100, 500, \infty \]

Loss frame (open)

Gain frame (filled)
Predictions 1: Attention times

$\theta = 15$;

$t_1 = 0, 100, 500, \infty$

open: loss; filled: gain
Predictions 2: The size of the framing effect is a function of the time (limit) the DM has for making a choice.

\[ t_1 = 100; \]
\[ \theta = 10, 15, 20 \]

Loss frame (open)

Gain frame (filled)

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\[ t_1 = 100; \]
\[ \theta = 10, 15, 20 \]
open: loss; filled: gain
System 1
Preferences in System 1 follow PT.

System 2
Preferences in System 2 are a weighted average of PT and EU.
System 2: Weighted average of PT and EU

\[
\delta_2^* = w \cdot (V_G - V_S) + (1 - w) \cdot (EV(G) - EV(S)) \\
= w \cdot \delta_1 + (1 - w) \cdot \delta_2.
\]

- Qualitative predictions remain as before.
System 1
Preferences in System 1 are modeled according to a Motivational function weighted by Willpower strength and Cognitive demand (MWC). (Loewenstein et al., 2015)

System 2
Preferences in System 2 are modeled according to EU.
Motivational function $M(x, a)$; $a$ captures the intensity of affective motivations

Function $h(W, \sigma)$ reflects the willpower strength $W$ and cognitive demands $\sigma$. 
\[ M(x, a) = \sum w(p_i) v(x_i, a) \]

- \( w(p) \) is a probability-weighting function

\[ w(p) = c + bp \text{ with } w(0) = 1, w(1) = 1, \text{ and } 0 < c < 1 - b \]

- \( v(x, a) \) is a value function that incorporates loss aversion

\[ v(x, a) = \begin{cases} a u(x) & \text{if } x \geq 0 \\ a\lambda u(x) & \text{if } x < 0 \end{cases} \]

- \( h(W, \sigma) \) is not specified but meant to be decreasing in \( W \) and increasing in \( \sigma \).

\[ V(x) = \sum u(x_i) + h(W, \sigma) \cdot \sum w(p_i) v(x_i, a) \]
WMC: static-deterministic model

For predicting choice probabilities and choice responses time

→ dynamic-stochastic framework
System 1 and System 2

- MWC model assumes that both processes operate simultaneously.
- Therefore, System 1 and System 2 merge into a single drift rate and the two stages basically collapse into one single stochastic process.
- With $V_G$ and $V_S$ indicating the subjective value of the gamble and the sure option, respectively, the mean difference in valences (drift rates) in a gain and loss frame become

$$\delta_{gain} = V_G - V_{S_{gain}}$$
$$\delta_{loss} = V_G - V_{S_{loss}},$$
Guo, Trueblood, Diederich (2017), Psychological Science

- $2 \times (\text{time limits: no, 1 sec}) \times 2$ (frames: gain, loss)
- 72 gambles per condition, collapsed to 9 "gambles" per condition
- 8 catch trials per condition
- 195 participants
## Models tested

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of parameters</th>
<th>RMSEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PT_k$</td>
<td>8</td>
<td>1.43</td>
</tr>
<tr>
<td>PT with additional scaling factor</td>
<td>9</td>
<td>.44</td>
</tr>
<tr>
<td>Dual with PT and EU</td>
<td>10</td>
<td>.282</td>
</tr>
<tr>
<td>Dual with PT and weighted PT and EU</td>
<td>11</td>
<td>.283</td>
</tr>
<tr>
<td>$MWC_k$</td>
<td>10</td>
<td>1.40</td>
</tr>
<tr>
<td>MWC with additional scaling factor</td>
<td>11</td>
<td>.54</td>
</tr>
<tr>
<td>$MWC_{2stages}$</td>
<td>12</td>
<td>.49</td>
</tr>
</tbody>
</table>
Model accounts: Probabilities

Gain frame
PT
Dual
MWC
MWC₂
Loss frame

Pr(keep) 0.27 0.42 0.57
Pr(Gamble) 0.25

Amount given 32 56 79

Pr(Gamble) 0.25

Amount given 32 56 79

Pr(Gamble) 0.75
Model accounts: RT – no TP

Gain frame

PT

Dual

MWC

MWC₂

Loss frame

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Multi-stage models

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Model accounts: RT – TP

Gain frame

PT

Dual MWC

MWC₂

Loss frame

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Baron & Gürcay (2017) for moral judgments

\[ RT = b_0 + b_1 AD + b_2 U + b_3 AD \cdot U \]
Example 1, revisited

Influence of payoffs and discrimination with manipulated processing orders
Payoffs and discrimination

Diederich & Busemeyer (2006); Diederich (2008), with time constraints
Attributes and task

- Attribute 1: Payoffs (Same, Different, Neutral)
- Attribute 2: Lines (same, different)
- Task: "same"/"different" judgment
Presentation orders

Condition PL

\[ + \quad -5 \quad +5 \]

\[ = \quad \neq \]

Condition LP

\[ + \quad -5 \quad +5 \]

\[ = \quad \neq \]

Condition C

\[ + \quad +1 \quad -1 \]

\[ = \quad \neq \]
### All conditions

<table>
<thead>
<tr>
<th>Lines stimulus</th>
<th>Payoff matrix</th>
<th>Presentation order</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>different</em></td>
<td>Different (D)</td>
<td>Payoff–Lines (PL)</td>
</tr>
<tr>
<td><em>same</em></td>
<td>Same (S)</td>
<td>Lines–Payoff (LP)</td>
</tr>
<tr>
<td></td>
<td>Neutral (N)</td>
<td>Payoff/Lines (C)</td>
</tr>
</tbody>
</table>

2 × 3 × 3 factorial design
Predictions for Payoff–Line example

\( \mu_1 = .1, -1, 0 \) for payoffs D, S, N; \( \mu_2 = -0.05 \) for line same
**Order schedule**: The attribute sequence is either $(1, 2, 1, 2, \ldots)$ or $(2, 1, 2, 1, \ldots)$, depending on whether $k_1 = 1$ or $k_1 = 2$.

**Time schedule**: Geometric distribution

$$Pr(T = n) = (1 - r)^{n-1}r, \quad n = 1, 2, \ldots$$

Expected value: $E(T) = 1/r$
Two stages

- Parameter estimated simultaneously for conditions PL and LP
- Cross validation for condition C (some of the parameters same as for PL and LP)
M1: Payoffs processed first and then switch to the lines (PL and C).
M2: Lines processed first and then switch to the payoffs (LP and C).
M3: Payoffs processed first, switch to the lines and then switch back and forth between attributes (C).
M4: Lines processed first, switch to the payoffs and then switch back and forth between attributes (C).
M5: Start with any attribute and switch back and forth between them (C).
Payoffs and Lines: Parameters estimated for PL and LP

<table>
<thead>
<tr>
<th>Payoffs</th>
<th>$\mu_{PD}$, $\mu_{PS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lines</td>
<td>$\mu_{Ls}$, $\mu_{Ld}$</td>
</tr>
<tr>
<td>Attention switching</td>
<td>$r_{12}$, $r_{21}$</td>
</tr>
<tr>
<td>Decision bound</td>
<td>$\theta_{PL}$, $\theta_{LP}$</td>
</tr>
<tr>
<td>Residual</td>
<td>$R_{PL}$, $R_{LP}$</td>
</tr>
</tbody>
</table>

from 36 data points
Payoffs and Lines: Parameters estimated for C

Cross validation

Attention switching $r_{12}$ or/and $r_{21}$
Decision bound $\theta_C$
Residual $R_C$

from 18 data points
Results – Group 1

**Lines different same**

- **Choice probability (incorrect)**
  - Payoff-Lines
  - Lines-Payoff
  - Combined

- **Mean RT (correct) [s]**
  - Payoff-Lines
  - Lines-Payoff
  - Combined

- **Mean RT (incorrect) [s]**
  - Payoff-Lines
  - Lines-Payoff
  - Combined

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Results – Group 3

Lines *different* *same*

### Payoff-Lines

**Choice probability (incorrect)**

<table>
<thead>
<tr>
<th>Payoff-Lines</th>
<th>Lines-Payoff</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>S</td>
<td>N</td>
</tr>
<tr>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>0.6</td>
<td>0.8</td>
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**Mean RT (correct) [s]**

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<tr>
<td>D</td>
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<td>N</td>
</tr>
<tr>
<td>0.7</td>
<td>0.9</td>
<td>1.1</td>
</tr>
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**Mean RT (incorrect) [s]**

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### Payoff conditions

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</tbody>
</table>

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Best fit

For all groups:

- M3: Payoffs processed first, switch to the lines and then switch back and forth between attributes (C).
## Response patterns

<table>
<thead>
<tr>
<th>Group</th>
<th>Stimuli</th>
<th>PL Presentation</th>
<th>LP Presentation</th>
<th>C Presentation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>obs</td>
<td>pred</td>
<td>obs</td>
</tr>
<tr>
<td>1</td>
<td>PD Ld</td>
<td>f</td>
<td>+</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>PS Ld</td>
<td>f</td>
<td>+</td>
<td>s</td>
</tr>
<tr>
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<td>PN Ld</td>
<td>f</td>
<td>−</td>
<td>s</td>
</tr>
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<td>f</td>
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<td>f</td>
<td>+</td>
<td>s</td>
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<tr>
<td></td>
<td>PN Ls</td>
<td>s</td>
<td>+</td>
<td>f</td>
</tr>
<tr>
<td>3</td>
<td>PD Ld</td>
<td>f</td>
<td>+</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>PS Ld</td>
<td>s</td>
<td>+</td>
<td>f</td>
</tr>
<tr>
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<td>PN Ld</td>
<td>f</td>
<td>−</td>
<td>s</td>
</tr>
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<td>+</td>
<td>f</td>
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Kahneman D (2011) Thinking, fast and slow. Macmillan


